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THE MATHEMATICS TEACHER

EDITED BY
W. H. METZLER

ASSOCIATED WITH

EUGENE R. SMITH — HARRY D. GAYLORD
MAURICE J. BABB — WILLIAM E. BRECKENRIDGE

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THE REORGANIZATION OF THE MATHEMATICAL CURRICULUM IN SECONDARY SCHOOLS.*

BY PROF. ARTHUR D. PITCHER.

It is with considerable diffidence that I speak to you on the subject of the mathematical curriculum in the secondary schools. I am neither a teacher of secondary mathematics, nor an expert in the pedagogy of mathematics. However I have a permanent interest in the subject and its progress, and in the whole people to whom I believe the advancement of mathematics and mathematical instruction is fundamentally important. It is from this point of view that I speak. I cannot hope to give you an expert opinion on the curriculum, for I am not that kind of an expert. I can only register here my reaction on the subject in hand. It may well be doubted if my opinions are worth while. In fact I would not give them if I thought they would be accepted as given. I shall be thoroughly satisfied if I can emphasize a few well-known points of view, and perhaps stimulate a reasonable amount of criticism and thought.

The American secondary school has come to be a powerful factor in public education. It is indeed the people's college and the question whether it is doing what it should for them is a pertinent one. What then is the true end of the secondary school of to-day? I speak primarily of the public high school, and of

* An address delivered before the Association of Mathematical Teachers in New England at Manchester, New Hampshire, February 27, 1915.

those private schools who think of themselves as filling the same place in the educational system of the country as the public high school. No one can deny the right of a given private school to serve a special class of students. Such schools can hardly come within the scope of a general discussion of this kind. In particular I think we must agree with the oft-reiterated statement, that the purpose of the school of which we are speaking is not to prepare the student for college. I, for one, am quite ready and even anxious to agree to this, though I do not think it need be forgotten that some of these students will go to college. Also whenever I am called upon to make this agreement, and it is quite often these days, I want to follow it closely by calling attention to the fact that it is likewise not the mission of the public high school to prepare its students for the trades. It is the mission of the public high school to educate its students in the best sense of the word. I understand that to educate a student means to give him a knowledge of the culture of the world past and present, to school him into habits of accurate work, precise and clear thinking, in other words, to furnish him with the intellectual means for living his life as fully and appreciatively as possible. Incidentally this means that on leaving school he should have the power of further self-education. I do not think that during these years any direct account need be taken of the student's future need to earn money. This does not mean that I have any objection to vocational schools as such. On the contrary, I think that, in many cases, they are meeting a distinct need. If a boy of fourteen must learn a trade, or learn nothing, then it is much better for him to learn a trade. However it seems to me that we must think of this condition of affairs as unsatisfactory in a country like ours. We do not want to feel that we have any community or any class of people, whose children must be denied the privilege of a good high school education. In other words our educational ideals should be such, that if realized, the vocational school would no longer exist, or else its curriculum, like that of the college, would be based on a good secondary school education.

On the other hand I would not exclude any subject from the curriculum because it is practical. If it should be immediately

useful to the student, so much the better, if it is only properly educative. In attempting to secure any particular result by a given mass of material, I think it is always in order to inquire if the same result could be obtained by the use of material which at the same time would be of more practical value. If domestic science as a subject has all the educational values of Latin, then domestic science should be substituted for Latin in all courses for girls.

What then should be the content of the curriculum of the secondary school? In considering this matter seriously, I find myself in agreement with such men as Sachs, who in his book "The American Secondary School" gives English, a foreign language, history, mathematics, and science as five subjects which he calls the constants of the secondary school course. The course may contain besides, courses in domestic science, manual training, stenography, etc. This is, I think, a proper view of the needs of the curriculum. There are many who seriously question the foreign language, and many who maintain that mathematics should not be required, or at least that the requirements should be very much modified. There are others who believe in mathematics, but wish to change the content of the course, and the methods of instruction, to such an extent that you and I would scarcely recognize the proposed courses as mathematical. The criticisms of mathematics as part of the curriculum, come from so many sources and are so insistent that it behooves us to take account of them, and it may well be that in so taking account we will be lead to better things for mathematics. It is good for any one to take account of himself occasionally and to justify himself and his activities. Such an extensive and careful consideration of these matters as has been undertaken recently by a committee of this association can not fail to be of permanent value.

The report of this committee when it appears will doubtless contain a very complete bibliography. I want to refer here to an article in the MATHEMATICS TEACHER of June, 1913, entitled "What Mathematical Subjects should be introduced into the curriculum of the Secondary Schools?" by William Betz of Rochester, New York. Mr. Betz gives many references to literature that are valuable to one desiring to know current tendencies.

Now in considering the matter of mathematics as a part of the secondary school course, what principles should guide us? In the first place it seems to me that we should be very conservative. The educational values of a given subject, and of given material in that subject are so hard to determine, that that which in the light of experience has proved to be reasonably good, should not be lightly thrown away. It should be discarded only for new material whose value has been definitely determined and found to be superior. On the other hand ultra-conservatism is likewise undesirable. We should remember that there have been changes in the method and the material of mathematical instruction in this country and elsewhere, and that these changes have not been accompanied by serious calamity. We do not spend much time extracting cube roots, neither do we insist that geometry be presented according to the letter of Euclid. We all agree, I think, that the change is good.

This is surely no time to fail to take account of the mass of criticism against mathematics and its teaching. There is now available for the curriculum such a large amount of material, in such a great variety of subjects, that that which remains must amply justify itself. Mathematics may be good to train the student in persistency and accuracy, but if that is all it does it may well have to resign its place in favor of another that can do this and more. That material will remain which can supply the most appropriate and unique combination of values. Doubtless we agree that this is the material which ought to remain.

We must seriously ask ourselves then whether the material we present and our methods of presentation are suited to the best evaluation of the subject. There are doubtless many different matters of expedience that must enter into this consideration. Students have many different needs, but we want to meet these needs by as few different courses as possible. And these courses must fit into each other in such a way that a student can change his mind as to his ultimate destination with a minimum loss.

Let us first ask ourselves then what are the values that we expect from the courses in mathematics. Not long ago the answer would have been: training of the logical powers. In the light of modern psychology we cannot lay too much em-

phasis on this, though lately even the psychologists seem to be admitting that the subject has a certain value of this sort. However at present we attach other values to mathematics. As I see it, mathematics has in the first place an historical value. It gives the methods and facts of a subject which has been at the foundation of progress in the physical sciences, and which to-day is more useful than ever. It certainly does give a training in careful accurate persistent work, the effect being upon the general character of the student, perhaps, rather than upon his logical faculties. The subject has a certain æsthetic value. It possesses certain elements of beauty that are unique. It gives to the student an opportunity to use his creative faculties in a way that scarcely any other subject does. Mathematics also has a tremendously practical value in that it is indispensable to the comprehension of much of the scientific learning of the day, and in that it is at the basis of all forms of engineering science. Thus we think of at least five values that attach to the subject, (*a*) the historical value, (*b*) the character-building value, (*c*) the æsthetic value, (*d*) an aid to understanding science, (*e*) the utilitarian value. Now how far do our present curriculum and methods of teaching, realize these values? I think we must see that so far as the teaching is concerned it does not do what it should. This is not possible with the present training that our teachers have. How can a teacher realize the historical value unless he himself is thoroughly acquainted with the history of mathematics and the allied sciences? How can he realize the æsthetic value unless he has pursued the study of mathematics long and successfully, and is indeed constantly interested in furthering his knowledge of the subject? He must himself be finding things that are new to him and be studying the approaches to problems for him unsolved, if he is to appreciate the interest and value of this sort of thing to the student. How can the teacher point with convincing confidence to the vast fields of application of the subject unless he has intimate acquaintance with the various fields of higher mathematics and a fair knowledge of at least physics and astronomy? It may well be that, to realize the logical training from geometry, all the equipment that is necessary on the part of the teacher is a keen mind and a familiarity with the subject

itself. Such a teacher may even do much more, but if his knowledge stops here his teaching must fall far short of completeness. You will understand that I am criticising no one in particular. Teachers of mathematics, so far as I know them, are very conscientious and progressive, and are doing a very great deal with the material at their disposal. I am merely emphasizing what I consider to be a fundamental weakness in our present system. However it will be noted here that I am indicating a training as complete as that of most college and university teachers, and I think we shall have made great progress when we see that such a training is none too good for the teacher in the secondary school. We should look forward to the time when every teacher of mathematics has a good graduate training in mathematics. The college and university teacher would add to this a training in mathematical research, and the secondary teacher a training in the history and pedagogy of mathematics. However in the immediate sequel much could be done if only the mathematics in the secondary schools were taught by those who in college had at least a reasonable interest in the subject and who studied it there to some extent. Too often, especially in the smaller schools, the mathematics is taught by someone with no training or no fundamental liking for the subject, someone perhaps who is trained in English and history but who is given the mathematics because that is a subject anyone can teach. That teacher deserves only sympathy. I feel sure that much of the present criticism of mathematics in the schools comes, not from the subject-matter of the courses, but from the methods of teaching. Here then I think should be our first answer to criticisms at present directed against the curriculum, especially those criticisms that come from educators and school officials. We should say to them: "Do what you can to see that the conditions for the teaching of mathematics are as favorable as possible, and see to it that at least only those teach mathematics who want to do so. We will see to it that those who want to teach mathematics do so with a steadily improving equipment and with a greater efficiency. We can do this much better, if you will see to it that we do not teach more hours per week than is consistent with the best results. We are not sure that the subject matter of our courses is chosen to the best ad-

vantage but we do not think it should be changed except after careful study and experiment. This study and experiment we ourselves have in hand, and we shall be among the first to recommend change, when it is clear what change is desirable." We might add to this that in any case we can easily see to it that the student who passes our courses works with some steadiness and persistence, and that this might not be true in courses which some propose instead of mathematics. But this might be both irrelevant and impertinent.

As to the improvement of the equipment of the present teacher, that is something that every one of us can help to accomplish by improving himself. Some can make use of leave of absence, or of summer vacations to attend institutions of learning. Those who cannot do this can at least make use of the increasing volume of good literature available, to increase both technical and professional equipment.

But how shall we proceed in the matter of the improvement of the curriculum? We must depend upon the schools of education in the larger universities where educational experts, with the cooperation of skilled teachers, can experiment under proper conditions with new methods and new material which seem to be more desirable than those in current use. Perhaps, also, such experimentation may be carried out in the larger schools where there are many teachers and where proper classes for experiment may be formed.

Under present conditions then, with our ideals of secondary education, where is the curriculum at fault? What changes should we be thinking about? How shall we go about finding suggestions as to possible desirable changes? In the first place we can take account of the experience of other people in other countries. While it is proper and expedient to make use of the experience of other countries, we should in no case try to graft their system on our schools. Our conditions are peculiar to us, and our curriculum must be the one best calculated to meet our conditions. Many suggestions as to the improvement of the curriculum come to us from our combined experience in the teaching of the present curriculum. It is my final purpose to-day to mention some changes in the curriculum suggested to me by my thought as to the various purposes of the secondary

school, and by my experience with students, and corroborated to some extent by the procedure in other countries.

In the first place, how about the relative proportion of the required and elective parts of the curriculum. I think the present proportion is certainly not far from right. It has been suggested that some sort of synoptic short course be given and that other mathematical courses be elective. I have no faith in this. Such a course would seem to me to be almost certainly valueless from every point of view. I do not see how it could be much more than a vague generality, which would even fail in requiring from the student a reasonable amount of wholesome effort. It is very hard to appreciate the facts and methods of mathematics without a certain mastery of technique. Teachers in college know how hard it is to give such courses even to students of considerable mathematical ability, and mastery of technique. No, I think that if the values of the subject are to be realized the required part of the course should consist of something more than half the whole. Then the question arises, is the material of the required course the best material. Here I am in serious doubt. With regard to algebra for example, I believe that the subject might be made somewhat less technical. I think the more difficult parts of it might be omitted, and emphasis placed more on the easy parts and their significance. Let us take for example the matter of factoring. Now it is quite desirable that the student should understand the general matter of factoring, and that he should be able to factor simple expressions. However I can not see that it is fundamental that he do more than this. If he meets a quadratic that he can not factor for example, it is not serious. Anyhow he will soon have at his disposal more powerful and direct methods which will produce the factors. Some little saving of time might be effected in this way, and at the same time the student would be relieved from an irksome task without apparent purpose. Then in geometry the work might be less formal. The introduction should make use of all the geometrical, mechanical, physical intuition that is available in the mind of the student. The introduction to the subject should be of an informal character. By means of drawing the student should be brought to comprehend the simpler figures and constructions in geometry,

and by degrees be made to see the necessity of argument to establish geometrical truth. Steps have been taken in this direction, and I know of a text-book soon to appear that in a very definite and admirable manner, makes the approach to the subject in this way. We may say that this is not geometry, but I think it is. After all a proof is a relative matter. The formal proofs of Euclid are not so perfect that we need feel that we must never depart from them. If by a more informal procedure we can allay the student's suspicion that geometry is really a very dry and uninteresting subject, suitable only as a taskmaster for young boys and girls, and bring him to see that it is really a very simple, common, every-day subject, concerning things that are very real to him, I think there will be no reason to fear that his later appreciation of the rigors of the subject will be below par. We are not living up to our privileges if we fail to make use of anything that the student already has in his equipment. I do not think, however, that we should go to the extent of making geometry all or even nearly all of this character. There should be a formal phase of the subject based on the conceptions that have been clarified by the preliminary work and on a set of unproved propositions which includes all those which to the student's mind need no proof. I know of nothing more confusing to youth or adult than to try to prove that for which there is no evident need of proof. I know that some of us have fixed in our minds a certain kind of text-book that defines in a supposedly definite fashion the entities of geometry and gives a certain very small number of axioms the truth of which every one admits and then starts off by proving some propositions equally evident. I believe, however, that just as this text is an improvement over Euclid, other texts and methods may furnish further improvement. It seems to me also that the formal part of the geometry should be as simple and easy as possible. Only the simplest propositions should be given, and the originals should be very numerous but very easy. The methods and virtues of precise formal reasoning may be exhibited as well by easy propositions as by difficult ones. The student may be better trained by doing a large number of easy originals than by failing to do a few hard ones. There should, in any case, be many examples of the disastrous results of false

procedure. For example in both algebra and geometry, the student should see by many examples that it is not safe to assume the truth of a given proposition, because it happens that using it as an hypothesis and reasoning from it in a perfectly correct fashion, he can derive a known conclusion.

In asking that fewer hard problems be given in algebra and geometry, I would have no notion of making the courses easier. What is lost in difficulty should be made up in thoroughness. If it worked out this way, the result would surely be a gain.

There are two serious changes which it seems to me might well be made in the required part of the curriculum. In the first place I think that the concept that we usually designate by the word "function" should be brought into the elementary algebra at the very beginning, and that every legitimate use of it should be made from then on. The graphical methods already in use in algebra would help to make this possible. This concept is one so useful everywhere in almost every aspect of human activity that I believe it would add a great deal to the value of the subject to the student. It is the very kind of thing that he will want to think about a great deal all his life. If he thinks about it clearly and accurately during his study of mathematics, it can not fail to be of use to him. That is, here is a place where the faculties of the student may be trained in the very form of activity that is to engage them in the sequel, independent of what he may do for a livelihood. Then also the function concept furnishes a unifying principle on which to build much of the elementary and advanced mathematics and helps to lend both concreteness and coherence to the subject. I do not mean that a fetish is to be made of the function concept, and that there should be an effort made to connect everything with it, whether it will or no. Such a fad tends to bring a notion into disrepute and properly so. Some years ago when graphical methods were being introduced into the secondary courses and into the text books, there was a desire on the part of some to make graphical methods account for everything. Now we are using graphical methods increasingly, and no doubt we all agree that they are good. But other methods are good and we shall retain them. We will be graphical when to be graphical suits us best, but we shall feel free to use other

methods when they seem to be best adapted to the matter in hand.

Then I think it would be very desirable to add to the required part of the curriculum the elements of trigonometry, not as a distinct subject, but as a part of algebra and geometry. I do not mean the whole subject of trigonometry as found in our text-books, but its simplest portions. For example in algebra, with the rectangular coordinate system at our disposal it is very easy and natural to define the trigonometric functions of the general angle and derive the simple relations amongst them. Simple trigonometric identities can be derived. In geometry use can be made of the functions of an angle, the law of sines and the law of cosines may be developed, and a part of the course in geometry may consist of the solution of triangles, right and oblique, by use of the natural functions. The student can be supplied with simple instruments and led to measure and compute heights and distances. I understand that all this needs to be very simple, but it can be simple and at the same time very interesting and instructive. From the standpoint of the various values to be expected from the required course in mathematics, it seems to me that it is a distinct mistake not to make use of this material, so easily available. The historical setting of the subject is very interesting, and very easily appreciated. As an aid to progress in other science and from a purely utilitarian viewpoint, the subject is very valuable. The concreteness of much of the subject matter and its power in solving certain types of interesting problems, can not fail to make a strong appeal to every student. It seems to me that from every point of view it would be desirable if possible to include this material in the required course. You will understand that I am not here especially considering my convenience as a teacher of college mathematics. If I did that, I might recommend that every moment possible be spent in drilling the student in the use of exponents, positive, negative, integral, and fractional; that the student be properly coached in the simplification of all kinds of complex and inhuman fractions, and that he be able to solve automatically any equation that may be reduced to the quadratic form. However, even if I were thinking of the matter most selfishly, I do not believe I would recommend such a

procedure. The boy or girl is likely to be most teachable who has from the beginning the best appreciation of the significance of the subject matter which he studies and his training in technique may fairly be distributed among the various stages of his student career.

Then how about the elective part of the course? Is the material for that well selected? It seems to me that here is our greatest weakness. The elective work is made up of three relatively distinct subjects each of them pursued for a half year, and one of them is itself rather an incoherent conglomeration. The material of this portion of the course should be chosen so as to best realize all the values of the subject, but with special attention to the role of mathematics as an aid to science and to its utilitarian value. It happens that, in my opinion, the material which would best realize these values would also best realize all the values which we have attached to the subject. With the graphical methods at our disposal, and with the function concept well developed in the required work, what could be more natural than to develop around these a unified course that might well include logarithms, some further study of trigonometry, selected portions of our present college algebra, and the very elements of analytic geometry and calculus. By the terms analytic geometry and calculus I do not signify the traditional college courses in these subjects. By analytic geometry I mean merely the simple and fundamental portions of that subject, such as would follow very readily and easily from the knowledge of the rectangular coordinate system which the student has already in his possession. By calculus I mean merely the notion of the slope of the tangent to a curve, the rate of change of one variable relative to another, the area under a curve, the evaluation of the limit of the sum by means of a definite integral. These ideas can all be illustrated by functions of a simple character. The functions chosen for illustration and for problems could be as simple as experience might indicate to be proper. Much of the technique of the subject, as now usually taught, could be avoided and yet a relatively clear notion of the character of the subject and the field of its application could be obtained. Plenty of simple problems are available for practise and they would be of great value if each one

of them were carefully and thoroughly studied with due emphasis on the geometrical and mechanical significance of each.

As I have already indicated, it seems to me that a course of this kind would be a considerable advantage over the present elective course. It is a more unified course and success in it would mean more continuous application and appreciation. It gets the student to a place where he can see to a much better advantage the dignity and far reaching power of mathematics. He can much better appreciate the genuine usefulness of the subject. If further schooling be denied him he is better able to study for himself. If his interests and work demand it, he can acquire by himself a considerable knowledge of mathematics or of physics, or of the elements of engineering. This is especially true when it is considered that students taking elective mathematics would likely take physics, and such other courses in science as were available. From the standpoint of the historical value of the subject of mathematics it is a shame to omit from a course which will be many a capable and interested boy's last chance at the subject, a topic which has had so important an influence on almost every aspect of world activity as the differential and integral calculus, even though it be admitted that only the most elementary and simple aspects of the subject can be included.

In view of the agitation in favor of omitting the incommensurable case in geometry one might hesitate to introduce the notion of the slope of a curve. Both make use of the notion of limit. However I think the slope of a curve is much the easier notion. It can be approached in a definite and concrete manner and the student will readily appreciate the character of the problem involved. I think that in the incommensurable case many students fail to see the character of the problem, or that there is any problem.

It will be noted that I have said nothing about solid geometry. I would not like to see solid geometry omitted. I would almost be willing to see it omitted if my other desires could be realized. However I see no reason why, since it is an elective anyhow, it could not be given as an extra subject for, say, two periods a week for a year. It seems to me to be not unwise even in the secondary school to permit students to specialize to a lim-

ited extent in the direction of their major interests. I would distribute the subject through a year because I feel its principal unique value, aside from a few formulas in mensuration, lies in the cultivation of the space intuitions. That would be best accomplished by a slower development through a longer period.

It will also be offered in criticism of my suggestions, that the material I have indicated will certainly require a longer time than now available for mathematics. This may be true. I am not sure of it. I know of no way to be sure except to work out the material in detail and give it several actual trials. I think there might be some point to such a criticism. However there are very many forces at work to extend the period of secondary instruction and it is rather clear to many people that in one way or another this period will be lengthened to five or six years. I believe we should lend our support to this as being desirable from every point of view. In my opinion this change is as likely and as imminent as is any serious change in our mathematical curriculum. Therefore the two things may be considered together. The longer period of secondary instruction would simplify many difficulties of the curriculum.

In conclusion, then, may I offer a brief résumé of what I have said.

1. Any serious change in the curriculum should be made only after most careful consideration based on skilled experimentation. In a matter of this kind leaders in education ought to be truly conservative.

2. Much of the criticism against the curriculum will disappear in the face of an increasingly careful and thorough preparation of teachers.

3. In spite of our conservatism we should be awake to every possibility of improvement, realizing that only thus will there ever be even a suggestion for improvement. Our conservatism should operate only to prevent the final adoption of any reform measure without due consideration and experimental evaluation.

4. A short synoptic course at beginning of high school is not feasible.

5. The curriculum at present is very good. Of possible changes that we may look forward to, I think that those most deserving of consideration are:

(*a*) The omission of some of the more difficult technique from algebra.

(*b*) The informal, intuitional approach to plane geometry with perhaps some abbreviation of that subject by omitting unnecessary propositions and difficult originals.

(*c*) The introduction of the function concept near the beginning of the required course and its use in the sequel both for its own sake and as a basis of unification.

(*d*) The introduction of the elements of trigonometry into the required course.

(*e*) The unification of the main part of the elective course around the function concept and the notion of the coordinate system including the elements of analytic geometry and calculus.

Some of these changes may of course be incorporated into the present curriculum without serious alterations. Some of them are already being incorporated. Others would require great change in point of view and method and might require a longer period of instruction than is now allotted to mathematics. While after considerable reflection I believe all the changes suggested would be desirable, I also believe that no one of them or any other should be adopted until by actual experiment, by skilled and open-minded teachers, it has been proved to be really desirable.

DARTMOUTH COLLEGE.

MATHEMATICAL PREPARATION DESIRED FOR HIGH SCHOOL PHYSICS.*

By Mr. Clifford S. Griswold.

Physics teachers are not particularly united either in their method of teaching, in the selection of topics, in emphasis on the various parts of the topics, or even the object to be attained by their teaching, but there is one point on which they all agree: that the relation between mathematics and physics is very unsatisfactory.

Pupils who have done well in their mathematics seem to be all at sea when they reach physics. Principles learned and applied with facility in the problems and equations of mathematical textbooks are beyond their power to apply in this new field. Computation is labored and beset with errors,—many simple operations requiring repeated explanation notwithstanding the training in arithmetic.

In geometry, lines, surfaces, angles, etc., are studied constantly, and their relations thoroughly investigated, but not a pupil enters the physics laboratory with any idea of accurate measurement of these familiar things, nor has he first-hand knowledge that these actually demonstrated facts are true in practice.

The physics teacher, given what seems to him a very meagre allowance of time in the school course to teach the physics that he is required and wants to teach, feels that it is quite impossible for him to take the time to teach the pupil the mathematics he doesn't know, and to adapt the mathematics he does know, in short, to make him over mathematically.

He has before him two alternatives, either to get along without mathematics and pay more attention to lecture table demonstration, dispensing with the problem book, or to change the mathematics teaching so that it shall coördinate more helpfully

* Delivered before the Association of Mathematical Teachers in New England at Manchester, N. H., February 27, 1915.

with the physics. If he chooses the former he finds that after a time his progress is arrested and results can only be obtained by the aid of mathematics, for no real progress in science is possible without mathematics. Then comes the time that the teacher of physics comes up against the fact that the mathematics as received by the pupils in their early training is not suited for his use and he has now to consider the difficult matter of adapting or changing the early training so as to correlate more naturally with his subject. Now it is a truism in education that the concrete must come before the abstract and it is too often a matter of sad experience that many of our troubles come from requiring pupils to deal in abstractions before they are ready for them.

It is coming to be realized more and more that the old-fashioned home work, chores, farm work, hand work of all descriptions furnished a ground work of concrete experience, of which the value, importance and necessity have only lately been properly understood, as they provided a foundation of self education on which the structure of theory and abstract reasoning could be built surely and swiftly.

Mr. Edison is a brilliant example of the aid of concrete experience to mathematical or quantitative comprehension. With little knowledge of the theory of mathematics, beyond some arithmetic and almost no algebra, he yet can make quick complicated mathematical deductions by a sort of instinctive inference from his vast store of accumulated facts and experiences.

Few have this ability to the degree that Mr. Edison has, but every one knows that the more one's knowledge of practical relations of quantity, dimension, etc., is, the less likely one is to go astray in abstract theory, as he has always at hand the corrective presence of observed fact and experience to act as a check against possible error and misunderstanding.

It seems clear then that one of *the* great difficulties the mathematics teacher—that all teachers—meet with, is the lack of concrete experience on the part of the pupils.

The physics teacher was right to emphasize his laboratory work, his lecture table experiments, to insist upon examining, taking apart, putting together of fixtures, machines and apparatus, but he should have done this not as a ***substitute*** but as a ***foundation*** for mathematics.

I was asked by your president to discuss the subject of preparation in mathematics desirable for high school physics. In general, it seems to me that the best preparation might be formulated on the following lines, namely, to have the subject of mathematics conceived and presented to young pupils, as a means of quantitative expression of the relations of various laws, phenomena, etc., of which the pupils have had immediate experience in some striking concrete fashion, either through the use of models, diagram, actual measurement or experiment; rather than to conceive and present mathematics as an end in itself or to follow any such course as to permit of its being mistakenly so conceived.

That is, rather than begin the study of a new mathematical subject or process, as an unrelated, detached subject, to create conditions through the apparatus, model, or experiment which the new mathematical process is necessary to explain.

If mathematics could be so presented as a means to a definite and where possible physical and concrete end—and I know that this is an extremely difficult matter and one calling for extensive in the present method, program, and apparatus of mathematics teaching—physics could ask no more of mathematics teaching.

The pupil would come to physics not as to a strange subject offering violent contrasts and forced relations to his mathematics, but into a subject in which he feels his mathematics will permit him to penetrate more deeply. A subject of which he has already curiosity—and interest—stimulating glimpses, and which he already knows something about from the contributions that physics has made to the problems, situations, experiments, etc., for which he has had to study his mathematics to be able to deal with, and best of all, he will have learned that his mathematics is a tool for use and not an *accomplishment* for exhibition or passing examinations well.

How the mathematics course shall be planned in detail to meet this requirement I am afraid I am not wise enough to say. In the beginning at least, this question would be answered variously according to the opportunities for collecting this material or the particular bent of the mathematical teacher's mind.

At Booker Washington's school at Tuskegee, every new build-

ing put up furnishes an astonishing amount of original material for geometry.

At Groton, I loaned the simpler physical appartus to the mathematics teacher as one item of material for quantitative expression.

It would be well to introduce frequent exercises involving actual construction and accurate measurement,—which would lead naturally to drawings to scale—and show reason for accurate computation.

Make use of live, instructive, interesting problems. I find this in a recent algebra, "Five years ago a father was six times as old as his son and five years hence the father will lack five years of being three times as old as his son will be at that time. What are their ages?" Contrast that with the following, "Assuming the velocity of sound to be 1,100 ft. a second, find the distance of the point of discharge if 24 seconds elapse between seeing the lightning and hearing the thunder."

The first problem is not a good vehicle for the mathematics it illustrates, for it has been used to secure a bit of knowledge which in all probability the eager minded pupil would regard with considerable contempt.

The second problem might very well stimulate the pupil to try the experiment for himself. In any case it would be an introduction to a line of phenomena which he would therefore be willing and able to learn more about later.

Popular knowledge and reading abounds in references to *candle power, amperes, velocities, temperature, horse power, machines, lenses, mirrors,* etc., in which all pupils are interested and an understanding of which is most meagre without some mathematics—furthermore the work of *engineers, electricians, surveyors, navigators, mechanics, contractors,* etc., contain materials for countless problems on subjects of familiar interest.

The study of graphs is useful in many ways to the mathematics teacher, in giving the pupil a better understanding of linear, simultaneous and quadratic equations. I have known many cases of boys failing to grasp the principles of simultaneous equations until they had seen the graphs *intersect* when it became perfectly clear to them.

The number and kinds of solution possible are more clearly

shown and understood through the graph than by theoretical proofs, which are more bewildering than helpful to the average pupil.

This knowledge of graphs is of great service to the pupil when he reaches physics, as experimental results in physics may often be expressed in the form of a graph.

Says John Perry: "The basis of all application of mathematics is the fact that any physical phenomenon which is directional, such as a force, a velocity, a stress, the flow of a liquid, etc., may be expressed by a straight line, and many other relations are expressed by curved lines, such as Boyle's law, capillarity, the pendulum, elasticity, the polarization of a voltaic cell, etc. Much of this data too can be used for plotting in the earlier mathematics.

It is generally true that the teacher of physics is obliged to teach whatever geometry is needed for his subject. The pupil beginning physics very soon encounters the composition of velocities and forces in the text, and soon again in the laboratory, but yet in spite of his earlier training in geometry, he has to be shown how to complete the parallelogram to find the resultant of the component forces.

Reflected motion, stability, curvilinear motion, pendulum, lever, inclined plane, wave motion, images in mirrors and lenses, and index of refraction of light are topics requiring a workable knowledge of constructional geometry, and these subjects again furnish data for the constructional problems of the earlier geometry, thus involving, in a purposeful way, work in lines, angles, perpendiculars, parallels, triangles, squares, and circles using cross-section paper, protractors, dividers, diagonal scale, and the metric rule.

GROTON, MASS.

SOME CLASS ROOM EXPERIENCES IN TEACHING GEOMETRY.

By G. I. Hopkins.

As a sort of prelude I beg leave to offer a few thoughts suggested by an address to this association at the previous meeting wherein the utilitarian, or practical phase of the study of geometry was not only emphasized, but stoutly advocated, to the exclusion of the demonstrative.

This word "practical," it seems to me, has been more exploited and overworked than any other in the discussion of educational matters in recent years. Its advocates seem to convey the idea that no part of the education of a child is "practical" that cannot be put to immediate financial use. That if a pupil acquires a piece of information today at school, he should be able to market it to-morrow, and thus prove its utility. This word "utility" has become a sort of slogan by those, mostly specialists, who would limit the activities of the school to those things which the individual may at once commercialize.

Governor Hughes, at the Peary testimonial, said, "When the time comes that knowledge will not be sought for its own sake, and without hope of gain, then, indeed, will the race enter upon its decadence." But may not a course of study be useful even though it does not contribute to the weekly pay envelope? I think I agree with Mr. T. J. McCormack who has met this question, so far as mathematics is concerned, without dodging, and in a very frank manner. He says: "Not one tenth of the graduates of our high schools ever enter professions in which their algebra and geometry are applied to concrete realities; not one day in 365 is a high school graduate called upon to apply, as it is called, an algebraic or geometrical proposition,—Why, then, do we teach these subjects, if this alone is the sense of the word 'practical'? To me the solution of this paradox consists in boldly confronting the dilemma, and saying that our conception of the practical utility of these subjects must be readjusted, and that we have frankly to face the truth that the 'practical' ends we

seek are, in a sense, ideal practical ends; yet such as have, after all, an eminently utilitarian value all their own, in the intellectual sphere."

At the same time I do not wish to be understood as attempting or desiring to belittle the importance of the practical applications of the ascertained truths of geometry. But the number of such truths susceptible of this application is surprisingly small, and the pupil must either accept them on faith or demonstrate them for himself. If the former, then he is not studying geometry, but simply doing some computing. If the latter, then he must begin at the beginning and take some logical sequence of theorems and prove them one by one until he reaches those he wishes to use. Anything less than this would result in a pseudo demonstration, partly mechanical and partly guess work.

The opponents of demonstrative geometry have declared, with considerable emphasis, that a boy who has taken such a course will be unable to think more logically when confronted with the problems of daily life, but I think he can and will. I heard one of them say not long ago that the study of geometry would not enable a man as a judge or juryman to sift and weigh evidence any better. In reply to this, a lawyer friend of mine, now a member of the supreme court of the state, said to me that he made it a practice to read Euclid's "Elements" through, at least, once a year, for the discipline it gave him in logical reasoning. Dr. Eugene Smith says: "Demonstrative geometry offers the best developed application of logic that we have, or are likely to have, in the school course; and to the pupil it offers a concrete example of close and exact reasoning that is not afforded by any of his other studies,—and it is extremely probable that the man who has had some drill in syllogisms, and who has learned to select the essentials in reaching his conclusions, has acquired habits in his reasoning that will help him in every line of work."

In the Perry Report Dr. Shaw said: "Let me pass on to say a word or two about the teaching of geometry as an academic training for general professional life. It has immense capabilities in that respect. If you consider how much of the effectiveness of an administrator depends upon the capacity for

coördinating appropriately a number of different ideas, precise accuracy of definition, rigidity of proof, and sustained reasoning, strict in every step, and when you consider what substitutes for these things nine men out of ten without special training have to put up with, it is clear that the man with the geometrical training has incalculable advantages." At a teachers' convention more than thirty years ago when this question of utility was being mooted, I heard an enthusiastic teacher of the classics exclaim, that he believed that a man who had to dig potatoes would accomplish that task better for having studied Latin.

That, we will agree, is an extreme case; but if it be a fact that he works more contentedly, and while performing his manual labor his mind becomes reminiscent, and conjures up some of the beautiful passages of Virgil, of Cicero, and of Horace, and thus beguiles the weary hour, and is afforded a relief from the monotonous tedium of his uninteresting task, his time and energy spent upon the Latin classics certainly have a real value to him, though only an æsthetic value.

This leads directly to another phase of the subject, viz.: the pleasure one experiences in any line of study, and the satisfaction that comes from success in achievement. There is a peculiar joy, that often develops into a feeling of actual triumph, from the successful demonstration of a geometrical theorem that has been somewhat puzzling and baffling. And I know that I am not exaggerating when I say, that a considerable majority of the pupils in demonstrative geometry sometimes experience that exultant triumph, if the subject is taught in the right way, and the pupils are not robbed of their opportunity. To quote from Dr. Smith again, "To deny that our pupils derive this pleasure from the study of geometry is to confess ourselves poor teachers. It is surely partly æsthetic, the pleasure that comes from the symmetry of form, and the delight of fitting things together. As the pupil advances in the study of geometry this pleasure is augmented, and probably lies chiefly in the mental uplift that it brings, as he recognizes he is in contact with absolute truth, and the approach he is making to the Infinite."

There is just one more thought that is suggested in this con-

nection and which is here presented without elaboration. The study of geometry affords one of the most effective means for correcting habits of carelessness, inaccuracy, forgetfulness, slovenliness, and inattention, and this is by no means the least in importance.

But I do not need to argue this question further to the members of this association. The only phase of the subject on which we might differ would be the method of presentation. And here my class room experiences covering a period of thirty years must be my argument. The prevailing methods during that time can be classified as follows:

1. The memorizing of proofs with no original work, and little or no instrumental practice.

2. Memorizing the principal proofs, with exercises for original work both demonstrative and instrumental.

3. The syllabus method requiring all original work, or practically all.

4. That which may be termed the mechanical, which makes it merely a tool for the mechanic and artisan.

My first attempt was with a class in the second year in the high school, they having had elementary algebra the previous year. We used Wentworth's text book which had been copyrighted a few years previous. I became deeply interested in the work at once because of the exercises for original work, which was a new departure in geometry text, and although not an ideal one, it was so much superior to the prevailing texts that it deserved the immense success which it attained. The members of the class soon became imbued with the same enthusiasm which I felt over the exercises, especially so as they learned that many of those exercises were more difficult than the proofs printed for them to learn. When we had reached the subject of quadrilaterals I resolved to try an experiment. I had already satisfied myself that a large majority of the class could prove those theorems unaided that were printed for them to learn, I was also satisfied that the remainder would profit more by the attempt, even though but partially successful, than to memorize the written proof. Acting upon that conviction I collected the books from the pupils and arranged a syllabus for them, following closely, of course, Wentworth's text. The

success attained was beyond my fondest hopes, and since then we have used a syllabus of some kind. When one stops to think of it, why should the pupil be asked to read over another's proof of a theorem to memorize and then be asked to prove unaided a more difficult one?

Consistency, it seems to me, requires that he should, at least, have the opportunity for a trial at the former. I cannot see as there is a single argument in favor of the memorizing plan. I know it is expected that the instructor will try to devise various schemes to prevent the pupil from memorizing, like making a change in the form of the figure, or tipping it over, or changing the letters. When the pupil discovers this device to thwart him, he in turn prepares his defence, and so the time and energies of both pupil and instructor are wasted in a fruitless intellectual contest to see which shall outwit the other. How much more sensible and profitable must it be to give the pupil his opportunity for investigation unhampered. As well might the teacher of Virgil, for instance, place a metrical translation in the hands of his pupils to read, as for the teacher in geometry to discredit the pupil's ability by furnishing him a printed proof.

The results of using the syllabus method have been most satisfactory and beneficial to both pupil and instructor. To the pupil mainly for the increased power and independence acquired in all mathematical investigations, and for the pleasure and satisfaction that comes from successful achievement.

The interest developed in certain proofs has been somewhat remarkable. For example: as soon as we had finished that series of theorems relating to the measurement of angles, the pupils were given the following one.

"The angle formed by two tangents to the same circle is double the angle formed by the diameter drawn from one point of contact and the chord joining both points of contact."

The result was eight or ten different proofs, arising mostly from variations in construction. Subsequent classes have also had it assigned to them, and so far I have received more than *sixty* different demonstrations. Just suppose the author of a text book had provided a printed proof of that theorem for the pupil to learn. The Pythagorean theorem was also of much

interest, and I have been presented with, at least, three new proofs of that. When I entered college, no geometry was required for preparation, so that my course in plane geometry was in the freshman year, with Davies' "Legendre" the text book. Only once did our professor offer us anything outside the text. Only two members of the class succeeded in finding a proof, one of them myself, and it took all the spare time we could give to it for two months before we succeeded. Three or four years after I began to use the syllabus I gave that same one to the class and within a week eight members handed me a correct proof. I have always consoled myself with the belief that it was not because they were so much better scholars than I was, but because of the difference in the method of presentation.

Besides this independence of text book and development of power to think logically, there is fostered an attitude of keen criticism, both of phraseology and statement, so that slip-shod, illogical proofs are quickly detected. Some of them have been much quicker than myself to detect inconsistencies both in definitions and phraseology of theorems. One of the first to fall under the ban of criticism was the very elementary one, as it now appears in most of the recent text books, viz.: "The sum of all the angles about a point on one side of a straight line passing through that point equals two right angles." A boy went to the board, drew the diagram, having four lines drawn from a point in the line. In stating what was to be proven, he made such a long list of angles that he was interrupted before he had completed the list by objections from his classmates. Of course, the theorem means simply the sum of the five angles, but he had taken it literally, as he had a right to do. Had he been allowed to continue without objection he would doubtless have indicated the sum of the entire fourteen angles. The discussion resulted speedily in correcting that phraseology to conform to the intended meaning by inserting the word "consecutive" before the word "angles." A similar correction was made in the theorem relating to the sum of the angles about a point. They had got a pretty clear idea of what the term "adjacent angles" meant after a few illustrations, so it was not very surprising to find them critizing the phraseology of the

theorem, "If a side and two adjacent angles of one triangle are equal to a side and two adjacent angles of another, etc." It was a unanimous decision that there were no "adjacent angles" in a triangle; and they concluded that "two angles and included side," was entirely satisfactory and adequate.

About twenty years ago I was developing the subject of loci by the use of a moving point. During a pause a hand was raised, and the owner permitted to inquire. He said he didn't see how a point could move. That led to a general discussion, and he based his objection on the definition of a point; viz., "A point is that which has position, but not extension." He claimed that if a point moved it parted with the only property which it possessed, *i. e.*, position. We had to agree that motion could be predicated only of material things, but that the idea was useful, and that, although not strictly in accord with fact, mathematicians felt compelled to utilize it. That young man is now assistant professor of mathematics in one of our New England colleges.

When the term "straight angle" was first proposed, I thought if others were going to use it we might as well, although I did not favor it, and saw no real need of it. However, its use was allowed but not encouraged, and the consequence was, little was heard of it in the class. About ten years ago, however, the matter was brought to our notice in quite an emphatic manner. I had asked a lad what a triangle was, and he replied very promptly, "A triangle is a plane figure having three sides and six angles." He was asked to go to the board, draw his figure and explain. He readily complied; and his explanation was, "It has the three sides *AB*, *BC* and *AC*, the three oblique angles *A*, *B* and *C*, and the three straight angles *AB*, *BC* and *AC*." I have always suspected that he took that way to show how ridiculous the term "straight angle" was from his point of view, after the several illustrations of an angle. At any rate I came to a decision very quickly, and directed them to use their pencils and obliterate the paragraph relating to the straight angle.

There is another difficulty which some of the recent text books present that use that term, when the theorems are reached relating to exterior angles, the latter not having been defined. There has been no reference to straight angles in my classes

since then, and since we have the right angle, degree with its subdivisions, and radian, there is no need of another unit of angle measurement. About a year ago I sent letters of inquiry to about fifty professors of mathematics, both pure and applied, in New England and middle states, asking their opinion about the use of that additional unit, and without exception they declared their disapproval of its use, and one emphasized his objection by calling it "mathematical slang"! When the student takes up trigonometry he will, of course, be confronted with angles of 180°. But that is a very different thing from showing a pupil in his early 'teens a straight line and then in the next paragraph calling that same line an angle. That is as reprehensible as to state to the same young beginner that a straight line is a variety of curve, as occurs in a recent text-book. Words and phrases should convey some distinct and permanent meaning to the young beginner, and all inconsistent and perplexing phrases banished.

"Sect" and "ray" are terms that have also been introduced, but they serve only to confuse the beginner, however serviceable the expert mathematician may find them; and I agree with Dr. Smith when he says that the use of these terms is "pedantic." Those texts which make use of these terms in the plane geometry must see that there is the same necessity for a term to designate a terminated plane in the solid geometry; but they have not provided it, and so are open to the charge of inconsistency. In the words of a prominent member of our association, "Why introduce unnecessary elements of confusion when there are plenty of difficulties with the clearest notation?" My experience is that pupils have very little difficulty with the term "line," whether terminated or indefinite is meant.

Pupils, as a rule, do not like the indirect method of proof. Because of this I have offered special inducements to pupils to discover direct proofs of those theorems of which we knew only the indirect. The result is three or four such direct proofs. I doubt very much if this could have been accomplished without the syllabus plan.

Pupils also dislike many corollaries, and, I must confess, I agree with them. We must not forget that what may rightly be an easy corollary to the mature and expert mathematician,

is by no means such to the young beginner in his early 'teens groping his way. And it is for these that the text book should be written. One recent text has carried this matter of corollaries to an absurd extent by giving *thirteen* corollaries to the theorem about the angle sum of a triangle. In view of all this evidence, which is the true, scientific, pedagogical, method of presenting the subject? Instead of printing the proofs why not allow the pupil to utilize his time and energies in the endeavor to originate them himself? This question is particularly pertinent when we consider the number of essential theorems compared with the number of exercises that are exercises only. The first six books of Euclid, excluding the fifth which treats of proportion, contains 148. Dr. Eugene Smith thinks a syllabus of 140 will suffice. The Harvard syllabus contains 102, but without any problems in construction. The syllabus of our association contains 110, with the suggestion that it could profitably be increased. Why not decide upon a syllabus of about 150 or 160, and let the pupil try his skill and mettle on those, rather than print these for him to learn and then require of him the proofs of 300 or 400 exercises that he can never make any use of?

It is difficult for the teacher, to whom the subject is familiar, to appreciate the standpoint of the beginner, especially in mathematics, unless he is constantly on the alert. The tendency is to get impatient over the seemingly petty difficulties which they encounter. But we must remember that those things which have ceased to be difficulties to the mature and experienced mind, through long familiarity, may be, and very often are, real and present difficulties to the beginner.

We should not get discouraged over the slowness with which they get under way, for their reasoning faculties develop slowly, and they must be allowed time to ponder. Let us not forget that the chief aim in the study of mathematics should be "to strengthen and develop the pupil's faculties, and to supply a method of reasoning applicable to other subjects and to other conditions, rather than to furnish him with an instrument for solving problems, believing that the shortsightedness of a narrow education, of an education that teaches only machines to a prospective mechanic, and agriculture to a prospective farmer,

and cooking and dress-making to the girl, and that would exclude all mathematics that is not utilitarian in the narrow sense, cannot endure."

And above all else in importance, let us not rob the pupil of his opportunity by placing before him the printed proof until he has, at least, had a chance, to struggle with it. Even then, a suggestion or two, or a well directed question, will afford him all the assistance he needs, in a large majority of cases. No permanent benefit can come to the pupil's mind when all he has to do is to yield ready assent to the statements supplied by another. Mental power is acquired only by a mental struggle, and this is impossible for the beginner in geometry when the proofs are furnished him, since the opportunity for the requisite mental struggle is denied him. Consistency demands that many of the so-called exercises be eliminated, and the proofs of those in the regular sequence be suppressed. In no other way can the pupil be given his rightful opportunity for both the pleasure and profit that comes from a rational study of geometry.

Since the above address was given there has come to my notice in the MATHEMATICS TEACHER an announcement of a book entitled "Inductive versus Deductive Methods of Teaching," by W. H. Winch. In this announcement occurs the following statement: "In five different schools of London, attended by children of different social classes, a series of experiments were made to test the relative values of inductive and deductive methods of teaching as applied to geometrical definitions.

"It was found that in three of the schools tested those taught deductively could reproduce precisely what they had been taught better than those taught inductively. In the other two the results were about even. In testing to find out which of the two methods gave the better results when the children were tested on new material it was shown that in all five schools those taught inductively did better work. This would seem to be a strong argument in favor of the syllabus method of teaching geometry."

HIGH SCHOOL,
MANCHESTER, N. H.

REPORT OF THE COMMITTEE OF BIBLIOGRAPHY.

The committee has written to the publishers of secondary school texts, the colleges, the national and state departments, and the associations of teachers of mathematics, inquiring as to their publications. The list of books accompanying this report has been prepared from the material sent in answer to these requests, and everything which might be of interest to a teacher of secondary mathematics was included unless it had already been listed in the "Bibliography of the Teaching of Mathematics" published by the United States Bureau of Education as Bulletin No. 503. Books omitted from the publishers' latest catalogs are not included. It has not seemed necessary for the purpose of this report to index its various parts, as any title wanted can easily be found under its appropriate sub-head.

Corrections and suggestions will be welcomed.

EUGENE RANDOLPH SMITH,
Chairman.
WALTER ROBERTS,
CLARENCE P. SCOBORIA,
GEORGE F. WILDER.

PART I.

Text Books.

Topics: Algebra; Geometry; Trigonometry; Miscellaneous.

The following abbreviations, enclosed in parentheses, will be used to denote the publishers indicated:

(A.) American Book Company.
(A.B.) Allyn and Bacon.
(Ap.) D. Appleton & Co.
(B.) A. S. Barnes & Co.
(C.) The Century Co.
(Ca.) Cassell & Co.
(Ch.) University of Chicago Press.
(C.P.) Cambridge Press.
(Cr.) E. S. Crawley, University of Pennsylvania.

(D.) E. P. Dutton & Co.
(G.) Ginn & Co.
(Go.) R. B. Goff, Fall River, Mass.
(H.) D. C. Heath & Co.
(H.H.) Henry Holt & Co.
(H.N.E.) Hinds, Noble and Eldredge.
(J.) B. F. Johnson Publishing Co.
(Jo.) G. W. Jones, Cornell University.
(S.J.) S. T. Jones, Gunter, Texas.
(L.) J. B. Lippincott Co.
(L.B.) Little, Brown & Co.
(L.G.) Longmans, Green & Co.
(M.) Chas. E. Merrill Co.
(Mac.) MacMillan & Co.
(M.H.) McGraw Hill Book Co.
(No.) H. H. Norris, Cornell University.
(O.) Oxford Press.
(Pu.) G. P. Putman's Sons.
(R.) R. H. Rivenburg, Hightstown, N. J.
(Ru.) Rutgers College.
(S.) Chas. Scribner's Sons.
(Sa.) B. H. Sandborn & Co.
(Sb.) Sibley & Co.
(So.) Christopher Sower Co.
(S.B.) Silver Burdett & Co.
(S.F.) Scott, Foresman & Co.
(V.) D. VanNostrand & Co.
(W.) John Wiley & Sons.

Algebra.

The fact that an algebra is published in parts is indicated by the numbers I and II following the title. If one part only is published separately, its number alone is used. The word (Key) following a title indicates that it is published both in text-book and key form.

1. Aley & Rothrock ..... The Essentials of Algebra (S.B.)
2. Atwood, G. E. ..... Standard School Algebra (S.B.)
3. ——— ..... Higher Algebra (S.B.)
4. Ball, W. R. ..... Elementary Algebra (C.P.)
5. Beman & Smith ..... Academic Algebra (G.)
6. ——— ..... Elements of Algebra (G.)

7. Bowser, E. A. ..... Academic Algebra (H.)
8. ——— ..... College Algebra (H.)
9. Boyd, J. H. ..... College Algebra (S.F.)
10. Bradbury, W. F. ..... Elementary Algebra (D.)
11. Bradbury & Emery ..... Algebra for Beginners (D.)
12. ——— ..... Academic Algebra (D.)
13. Brooks, E. ..... Elementary Algebra (So.)
14. ——— ..... Standard Algebra (So.)
15. Brown, T. K. ..... Elementary Algebra (A.)
16. Colaw & Powers ..... School Algebra (J.)
17. ——— ..... Elementary Algebra (Key) (J.)
18. Collins, J. V. ..... Practical Algebra (I and II) (A.)
19. ——— ..... Practical Elementary Algebra (A.)
20. Davison ..... Algebra for Secondary Schools (C.P.)
21. Dupuis, N. F. ..... The Principles of Elementary Algebra (Mac.)
22. Durell, Fletcher ..... Introductory Algebra (M.)
23. ——— ..... School Algebra (M.)
24. Evans, G. W. ..... Algebra for Schools (H.H.)
25. Downey, J. F. ..... Higher Algebra (II and Complete) (A.)
26. Fairbank & Hebden ..... Elements of Algebra (D.)
27. Fine, H. B. ..... College Algebra (G.)
28. Fisher & Schwatt ..... Rudiments of Algebra (Mac.)
29. ——— ..... Secondary Algebra (I and Complete) (Mac.)
30. Fite, W. B. ..... First Course in Algebra (H.)
31. French & Osborn ..... Elementary Algebra (C.P.)
32. Gideon ..... Model Elementary Algebra (Key) (H.N.E.)
33. Gilbert, J. H. ..... Algebra Lessons for Supplementary and Review Work (3 Parts) (H.)
34. ——— ..... Practical Lessons in Elementary Algebra (Mac.)
35. ——— ..... Complete Algebra (Mac.)
36. Gorse, F. ..... A School Algebra Course (3 Parts) (C.P.)
37. Graham, R. M. ..... Elementary Algebra (L.G.)
38. Griffin, W. M. ..... Grammar School Algebra (A.)
39. Hall & Knight ..... Algebra for Colleges and Schools (Mac.)
40. ——— ..... Elements of Algebra (Mac.)
41. Hawkes, H. E. ..... Advanced Algebra (G.)
42. Hawkes, Luby & Touton .. Complete School Algebra (I and II) (G.)
43. Hedrick, E. R. ..... Algebra for Secondary Schools (A.)
44. Hopkins & Underwood ... A First Book in Algebra (Mac.)
45. Hull, G. M. ..... Elements of Algebra (A.)
46. ——— ..... Complete Algebra (Key) (A.)
47. Jocelyn, L. P. ..... High School and Academic Algebra (A.)

48. Keller, S. S. .....Algebra and Trigonometry (I, II) (V.)
49. Kent, F. C. .....A First Course in Algebra (L.G.)
50. Lilley, G. .....The Elements of Algebra (Key) (S.B.)
51. ——— .....Higher Algebra (S.B.)
52. Lippincott's .....Elementary Algebra (Key) (L.)
53. Long & Brenke .....Algebra, First Course (C.)
54. ——— .....Correlated Mathematics for Secondary Schools (C.)
55. MacDonald, J. W. .....A Primary Algebra (Key) (A.B.)
56. Marsh, W. R. .....Elementary Algebra (Key) (S.)
57. Marsh & Ashton .....College Algebra (S.)
58. Metzler, Roe and Bullard College Algebra (L.G.)
59. Milne, W. J. .....Grammar School Algebra (A.)
60. ——— .....Elements of Algebra (Key) (A.)
61. ——— .....First Year Algebra (Key) (A.)
62. ——— .....High School Algebra (Key) (A.)
63. ——— .....Standard Algebra (Key) (A.)
64. ——— .....Academic Algebra (Key) (A.)
65. ——— .....Advanced Algebra (Key) (A.)
66. Morris, Richard .....Selected Chapters in Algebra (Ru.)
67. Nicholson, J. W. .....School Algebra (Key) (A.)
68. Palmer, C. I. .....Algebra with Applications (M.H.)
69. Patterson, W. E. .....School Algebra (I, II, III) (O.)
70. Ray, Jos. .....New Elementary Algebra (A.)
71. ——— .....Higher Algebra (A.)
72. Rietz & Grathorne .....College Algebra (H.H.)
73. Robinson, H. N. .....New Elementary Algebra (Key) (A.)
74. ——— .....New University Algebra (Key) (A.)
75. Sabin & Lowry .....Elementary Lessons in Algebra (Key) (A.)
76. Sanford, S. P. .....Elementary Algebra (Key) (A.)
77. ——— .....Advanced Algebra (Key) (A.)
78. Schultze, A. .....Elementary Algebra (Mac.)
79. ——— .....Elements of Algebra (Mac.)
80. ——— .....Advanced Algebra (Mac.)
81. Sensenig, D. M. .....Elementary Algebra (Key) (A.)
82. ——— .....Advanced Algebra (Key) (A.)
83. Sheldon, E. A. .....Elements of Algebra (Key) (A.)
84. ——— .....Complete Algebra (Key) (A.)
85. Slaught & Lennes .....First Principles of Algebra (I, II) (A.B.)
86. ——— .....High School Algebra (A.B.)
87. Smith, Charles, revised by Stringham .....Elementary Algebra (I) (Mac.)
88. Smith, D. E. .....Algebra for Beginners (G.)
89. Somerville, F. H. .....First Year Algebra (A.)
90. ——— .....Elementary Algebra (A.)
91. Stone & Millis .....Essentials of Algebra, Brief Course and Complete (Sa.)

92. ——— .....Elementary Algebra (I, II, C) (Sa.)
93. Tanner, J. H. .....Elementary Algebra (Key) (A.)
94. ——— .....High School Algebra (Key) (A.)
95. Taylor, J. M. .....Elements of Algebra (A.B.)
96. ——— .....Academic Algebra (A.B.)
97. Venable, C. S. .....Easy Algebra (A.)
98. Wells, Webster .....Algebra for Secondary Schools (First Course, Second Course) (H.)
99. ——— .....Textbook in Algebra (H.)
100. ——— .....Essentials of Algebra (H.)
101. ——— .....Academic Algebra (H.)
102. ——— .....New Higher Algebra (H.)
103. ——— .....Advanced Course in Algebra (H.)
104. ——— .....College Algebra (Parts I, II) (H.)
105. ——— .....University Algebra (H.)
106. ——— .....Higher Algebra (H.)
107. Wells & Hart .....First Year Algebra (H.)
108. ——— .....New High School Algebra (H.)
109. Wentworth, G. A. .....First Steps in Algebra (G.)
110. ——— .....School Algebra (G.)
111. ——— .....New School Algebra (G.)
112. ——— .....College Algebra (Revised) (G.)
113. ——— .....Elements of Algebra (G.)
114. ——— .....Elementary Algebra (G.)
115. ——— .....Complete Algebra (G.)
116. ——— .....Shorter Course in Algebra (G.)
117. ——— .....Higher Algebra (G.)
118. Wentworth & Smith .....Academic Algebra (G.)
119. ——— .....School Algebra (Parts I, II) (G.)
120. ——— .....Vocational Algebra (G.)
121. Wheeler, A. H. .....First Course in Algebra, Brief Edition; Complete (L.B.)
122. White, E. E. .....School Algebra (Key) (A.)
123. Wilson .....Elementary Algebra (Key) (H.N.E.)
124. ——— .....Elementary Algebra with Review in Algebra by Rubinstein (H.N.E.)
125. Young & Jackson .....Elementary Algebra (I, II) (Ap.)
126. ——— .....A High School Algebra (Ap.)

Problems in Algebra.

1. Atwood, G. E. .....Exercises in Algebra (S.B.)
2. Clay, C. M. .....Examples in Algebra (Mac.)
3. Davison .....Exercises from Algebra for Secondary Schools (C.)
4. Jones, G. W. .....Drill Book in Algebra (Jo.)
5. Leighton, L. R. .....Review of Algebra (Sb.)

6. McCurdy, M. S. .....Exercise Book in Algebra (H.)
7. Myers, G. W. .....Geometric Exercises for Algebraic Solution (Ch.)
8. Nunn, T. P. .....Exercises in Algebra and Trigonometry (L.G.)
9. Perrin .....Drill Book in Algebra (L.)
10. Peterson, S. J. .....Review and Test Problems in Algebra (H.)
11. Rietz & Grathorne .....Supplementary Exercises and Problems (H.H.)
12. Rivenburg, R. H. .....A Review of Algebra (A.)
13. Robbins, E. R. .....Algebra Reviews (G.)
14. Robbins & Somerville .....Exercises in Algebra (A.)
15. Rubinstein .....Review in Algebra (H.N.E.)
16. Tibbets, G. P. .....College Requirements in Algebra (G.)
17. Wentworth & Hill .....Exercise and Examination Manual (I, II) (G.)
18. Wheeler, A. H. .....Examples in Algebra (L.B.)

Geometry.

The words "plane," "solid," "combined," will be used to indicate plane geometry, solid geometry, and a combined plane and solid geometry, respectively.

1. Baker, A. L. .....Elements of Solid Geometry (G.)
2. Beman & Smith .....Plane; Solid; Combined (G.)
3. Betz & Webb .....Plane Geometry (G.)
4. Bowser, E. A. .....Plane; Combined (H.)
5. Bradbury, W. F. .....Elementary Geometry (D.)
6. ——— .....Academic Geometry, Plane and Solid (D.)
7. Brooks, E. .....Elementary Geometry (So.)
8. ——— .....Geometry and Trigonometry (So.)
9. ——— .....Plane; Combined (So.)
10. Bush & Clarke .....The Elements of Geometry (S.B.)
11. ——— .....The Elements of Plane Geometry (Key) (S.B.)
12. Candy, A. L. .....Plane and Solid Geometry (with Supplements) (H.)
13. Case, J. .....The First Six Books of the Elements of Euclid (L.G.)
14. ——— .....A Sequel to Above, Part I (L.G.)
15. Chauvenet .....Plane and Solid Geometry (L.)
16. ——— .....Plane and Solid Geometry (Key) (Byerly Edition) (L.)
17. Davies .....Legendre's Geometry and Trigonometry (A.)

18. ——— .....Legendre's Geometry (A.)
19. Dobbs, W. J. .....A School Course in Geometry (L.G.)
20. Durell, Fletcher .....Plane; Solid; Combined (M.)
21. Edwards, G. C. .....Elements of Geometry (Mac.)
22. Failor .....Plane and Solid Geometry (C.)
23. Ford & Ammerman .....Plane; Combined (Mac.)
24. Godfrey & Siddons .....A Shorter Geometry (Pu.)
25. Goff, Robert R. .....Syllabus of Geometry (Go.)
26. Gore, J. H. .....Plane and Solid Geometry (L.G.)
27. Hart & Feldman .....Plane; Solid; Combined (A.)
28. Hill, G. A. .....Geometry for Beginners (G.)
29. ——— .....Lessons in Geometry (G.)
30. Holgate, T. F. .....Elementary Geometry, Plane and Solid (Mac.)
31. Hopkins, G. I. .....Inductive Plane Geometry (H.)
32. Hornbrook, A. R. .....Concrete Geometry (A.)
33. Hull, G. W. .....Elements of Geometry (A.)
34. Jackson, W. H. .....Elementary Solid Geometry (L.G.)
35. Keigivin, H. W. .....Elements of Geometry (H.H.)
36. Keller, S. S. .....Plane and Solid Geometry (V.)
37. Langley & Phillips .....The Harper Euclid (In Parts) (L.G.)
38. Lyman, E. A. .....Plane; Solid; Combined (Key) (A.)
39. Low, D. A. .....Textbook in Practical, Solid, and Descriptive Geometry (I, II) (L.G.)
40. MacDonald, J. W. .....Principles of Plane Geometry (A.B.)
41. Milne, W. J. .....Plane; Solid; Combined (Key) (A.)
42. Morris & Husband .....Practical Plane and Solid Geometry (L.G.)
43. Petke, G. D. .....Plane Geometry (S.B.)
44. Phillips & Fisher .....Plane; Solid; Combined (Key) (A.)
45. Robbins, E. R. .....Plane; Solid; Combined (Key) (A.)
46. Robinson, H. N. .....New Geometry and Trigonometry (Key) (A.)
47. ——— .....Geometry (A.)
48. Sanders, A. .....Plane; Solid; Combined (Key) (A.)
49. Sanderson & Brewster .....A Geometry for Schools (Pu.)
50. Schmall & Shack .....Elements of Plane Geometry (V.)
51. Schoch, W. .....Introduction to Geometry (A.B.)
52. Schultze & Sevenoak .....Plane; Combined (Mac.)
53. Slaught & Lennes .....Plane; Solid; Combined (A.B.)
54. Smith, Eugene R. .....Plane; Solid; Combined (Developed by Syllabus Method) (A.)
55. Stone & Millis .....Plane; Solid; Combined (Sa.)
56. Venable, C. S. .....Elements of Geometry (A.)
57. Wells, Webster .....Elements of Geometry (Plane; Solid; Combined) (H.)
58. ——— .....New Plane; Solid; Combined (H.)

59. ——— .....Essentials of Geometry (Plane; Solid; Combined) (H.)
60. Welsh, A. H. .....Essentials of Geometry (S.F.)
61. Wentworth, G. A. .....Plane; Solid; Combined (Revised) (G.)
62. ——— .....Plane and Solid Geometry and Trigonometry (G.)
63. Wentworth & Hill .....First Steps in Geometry (G.)
64. Wentworth & Smith .....Plane; Solid; Combined (G.)
65. White, E. E. (by John Mac-nie) .....Plane; Combined (Key) (A.)
66. Young & Schwartz .....Elementary Geometry (H.H.)

Exercise Books and Tablets for Geometry.

1. Bates & Charlesworth .....Practical Mathematics and Geometry (V.)
2. Beman & Smith .....Geometry Tablet (G.)
3. Conant, L. L. .....Original Exercises in Plane and Solid Geometry (A.)
4. Edgett, G. L. .....Exercises in Plane Geometry (H.)
5. Estill, J. G. .....Numerical Problem in Geometry (L.G.)
6. Lyman, E. A. .....Geometry Exercises (H.)
7. Rivenburg, R. H. .....Geometry Tablet (R.)
8. Sykes, Mabel .....Source Book of Problems in Geometry (A.B.)
9. Wentworth, G. A. .....Geometrical Exercises (G.)
10. Wentworth & Hill .....Exercise Manual in Geometry (G.)
11. ——— .....Examination Manual in Geometry (G.)
12. Wright, D. S. .....Exercises in Concrete Geometry (H.)

Trigonometry.

The word "combined" will be used to indicate a complete plane and spherical trigonometry. (Tables) indicates that the book is published with and without tables.

1. Anderegg & Roe .....Plane Trigonometry (G.)
2. Bauer & Brooks .....Plane and Spherical Trigonometry (H.)
3. Bohannan, R. D. .....Plane Trigonometry (A.B.)
4. Bowser, E. A. .....Elements of Plane and Spherical Trigonometry (Tables) (H.)
5. ——— .....Five Place Logarithm Tables (H.)
6. ——— .....A Treatise on Plane and Spherical Trigonometry and Its Applications to Astronomy and Geodesy (H.)
7. Bradbury, W. F. .....Elementary Trigonometry (D.)
8. ——— .....Trigonometry and Surveying (D.)
9. Brooks, E. .....Plane and Spherical Trigonometry (So.)

10. ——— .....Geometry and Trigonometry (So.)
11. Byerly, W. E. .....Syllabus of Plane Trigonometry (G.)
12. Chauvenet .....Trigonometry (L.)
13. Conant, L. L. .....Plane; Combined (Tables) (A.)
14. Crawley, E. S. .....Elements of Plane and Spherical Trigonometry (Tables) (Cr.)
15. ——— .....Short Course in Plane and Spherical Trigonometry (Tables) (Cr.)
16. ——— .....Tables of Logarithms (Five Place) (Cr.)
17. Crockett, C. W. .....Plane; Combined (Tables) (A.)
18. ——— .....Logarithms and Trigonometric Tables (A.)
19. Durell, F. .....Plane; Combined (Tables) (M.)
20. ——— .....Logarithms and Trigonometric Tables (M.)
21. ——— .....Plane Trigonometry and Surveying with Tables (M.)
22. ——— .....Plane and Spherical Trigonometry with Surveying Tables (M.)
23. Durfee, W. P. .....The Elements of Plane Trigonometry (G.)
24. Gore, J. H. .....Plane and Spherical Trigonometry (Pu.)
25. Granville, W. A. .....Plane; Combined (Tables) (G.)
26. ——— .....Four Place Logarithmic Tables (G.)
27. Hall & Fring .....Plane; Combined (H.H.)
28. ——— .....Five Place Tables (H.H.)
29. Hall & Knight .....Elementary Trigonometry (Mac.)
30. Hussey, W. J. .....Logarithmic and Other Tables (A.B.)
31. Jones, G. W. .....Drill Book in Trigonometry (Jo.)
32. ——— .....Four Place Logarithms (Jo.)
33. ——— .....Five Place Logarithms (Jo.)
34. Keller, S. S. .....Algebra and Trigonometry (I, II) (V.)
35. Kenyon & Ingold .....Trigonometry (Tables) (Mac.)
36. Lambert & Foering .....Plane and Spherical Trigonometry (Mac.)
37. Lock, J. B. (Revised by Miller, J. A.) .....Trigonometry for Beginners (Tables) (Mac.)
38. Loomis, E. .....Plane and Spherical Trigonometry with Tables (A.)
39. ——— .....Tables of Logarithms (A.)
40. Lyman & Goddard .....Plane and Spherical (Tables) (A.B.)
41. Marsh & Ashton .....Plane and Spherical Trigonometry (Tables) (S.)
42. ——— .....Five Place Logarithmic Tables (S.)
43. Moritz, R. E. .....Plane; Spherical; Combined (W.)
44. Murray, D. A. .....Plane; Spherical; Combined (Tables) (L.G.)

45. ——— .....Essentials of Trigonometry and Mensuration (L.G.)
46. ——— .....Logarithmic and Trigonometric Tables (L.G.)
47. Newcomb, S. .....Elements of Plane; Combined (Tables) (H.H.)
48. ——— .....Essentials of Trigonometry, Plane and Spherical with Three or Four Place Tables (H.H.)
49. ——— .....Mathematical Tables (H.H.)
50. Nicholson, J. H. .....Elements of Plane and Spherical Trigonometry (Mac.)
51. Palmer, C. I. .....Trigonometry and Logarithms (M.H.)
52. Palmer & Leigh .....Plane Trigonometry with Tables (M.H.)
53. Phillips & Fisher .....Logarithms of Numbers (A.)
54. Phillips & Strong .....Plane and Spherical Trigonometry (Key) (Tables) (A.)
55. ——— .....Logarithmic and Trigonometric Tables (A.)
56. Robbins, E. R. .....Plane Trigonometry (A.)
57. Robinson, H. N. .....New Geometry and Trigonometry (A.)
58. ——— .....Trigonometry (A.)
59. Rothrock, D. A. .....Elements of Plane and Spherical Trigonometry (Mac.)
60. Smith, P. F. .....Four Place Logarithmic Tables (H.H.)
61. Taylor, J. M. .....Plane; Combined (G.)
62. ——— .....Five Place Logarithmic and Trigonometric Tables (G.)
63. Taylor & Puryear .....The Elements of Plane and Spherical Trigonometry (G.)
64. Wells, Webster .....New Plane; Combined (Tables) (H.)
65. ——— .....Same with Robbin's Surveying and Navigation (H.)
66. ——— .....Complete Trigonometry (Tables) (H.)
67. ——— .....Six Place Logarithmic Tables (H.)
68. ——— .....Four Place Logarithmic Tables (H.)
69. Wentworth, G. A. .....Plane and Solid Geometry and Plane Trigonometry (G.)
70. ——— .....Plane; Combined; Tables (G.)
71. ——— .....Plane; Combined; Surveying and Tables (G.)
72. ——— .....Plane and Spherical Trigonometry, Surveying and Navigation (G.)
73. Wentworth & Hill .....Logarithmic and Trigonometric Tables for Trigonometry, Surveying and Navigation (G.)
74. ——— .....Seven Place Tables for Trigonometry and Surveying (G.)

75. Wentworth & Smith .....Plane and Spherical Trigonometry (G.)
76. Whitaker .....Elements of Trigonometry (H.N.E.)

Miscellaneous.

1. Auerbach, Matilda .....Graphic Mathematics (A.B.)
2. Beman & Smith .....Famous Problems of Geometry (G.)
3. Breckenridge, Mersereau & Moore .....Shop Problems in Mathematics (G.)
4. Cajori, F. .....A History of Mathematics (Mac.)
5. Cobb, H. E. .....Elements of Applied Mathematics (G.)
6. College Entrance Examination Board .....Examinations in Mathematics (G.)
7. Cracknell, A. G. .....Practical Mathematics (L.G.)
8. Davidson, E. A. .....Linear Drawing and Projection (Ca.)
9. Davies & Peck .....A Mathematical Dictionary (B.)
10. Dooley, W. H. .....Vocational Mathematics (H.)
11. Fergusson, J. C. .....Percentage Trigonometry (L.G.)
12. Fine, H. B. .....The Number System of Algebra (H.)
13. Griffin, W. N. .....The Elements of Algebra and Trigonometry (L.G.)
14. Halsted, G. B. .....Metrical Geometry (G.)
15. Heath's Mathematical Monographs .....Famous Geometrical Theorems, W. W. Rupert
    On Teaching Geometry, Florence Miller
    Factoring, W. Wells
    The Triangle with Its Circles, H. W. Bruce
    Supplementary Algebra, R. L. Short
16. Henrici, O. .....Elementary Geometry—Congruent Figures (L.G.)
17. Jones, G. W. .....Some Proofs in Elementary Geometry (Jo.)
18. Jones, S. I. .....Mathematical Wrinkles (S.J.)
19. Lambert, P. A. .....Computation and Mensuration (Mac.)
20. Langley, E. M. .....A Treatise on Computation Methods for Contracting and Abbreviating Arithmetical Calculation (L.G.)
21. Loomis, E. S. .....Original Investigation: or How to Attack an Exercise in Geometry (G.)
22. Manning, H. P. .....Non-Euclidean Geometry (G.)
23. Marsh & Marsh .....Industrial Mathematics (W.)
24. McLachlan, N. W. .....Practical Mathematics (L.G.)
25. Morris, I. H. .....Geometrical Drawing for Art Students (L.G.)
26. Myers, G. W. .....First Year Mathematics for Secondary Schools (Manual) (Ch.)

27. ——— .....Second Year Mathematics for Secondary Schools (Ch.)
28. Newson, H. B. .....Graphic Algebra (G.)
29. Nichols, E. H. .....Elementary and Constructional Geometry (L.G.)
30. Norris, H. H. .....Syllabus of Mathematics (No.)
31. Palmer, C. I. .....Geometry with Applications (M.H.)
32. Peirce, J. M. .....Elements of Logarithms (G.)
33. ——— .....Mathematical Tables (G.)
34. Phillips & Beebe .....Graphic Algebra (H.H.)
35. Saxelby, F. M. .....An Introduction to Practical Mathematics (L.G.)
36. ——— .....A Course in Practical Mathematics (L.G.)
37. Short & Elson .....Secondary School Mathematics (Book I and Book II) (H.)
38. Smith, D. E. .....Certain Problems in Teaching Mathematics (G.)
39. Spencer, W. G. .....Inventional Geometry (A.)
40. Spooner, H. T. .....The Elements of Geometrical Drawing (L.G.)
41. Tibbetts, G. P. .....College Requirements in Algebra (G.)
42. Wentworth, G. A. .....Syllabus of Geometry (G.)
43. ——— .....Logarithms, Metric Measures, and Spherical Subjects in Advanced Algebra (G.)
44. Wentworth, McLellan & Glashan .....Algebraic Analysis (G.)
45. Young, J. W. A. .....Monographs on Topics of Modern Mathematics Relevant to the Elementary Field (L.G.)

PART II.

Publications of State Departments of Education, and of the the United States Bureau of Education, Relating to Mathematics in the Secondary Schools.

ALABAMA.

Rules and Regulations for the Government of the County High Schools. Montgomery, 1913. 16 p.

Contains course of study and lists of authorized texts.

ARKANSAS.

Course of Study for High Schools. Little Rock, 1913.

GEORGIA.

Manual for Georgia Teachers, Atlanta. 295 p.

A valuable general handbook on history, principles, and method in education, though with but little direct reference to secondary schools.

HAWAII.

List of Textbooks Prescribed for Use in the Public Schools in the Territory of Hawaii. Honolulu, 1912. 14 p.

IDAHO.

List of Textbooks Adopted by the State Textbook Commission. Boise, 1907. Pamphlet.

INDIANA.

Course of Study for Secondary Schools. Indianapolis.

KANSAS.

Course of Study for High Schools. Topeka, 1912. 208 p.

Contains outlines of courses in elementary algebra, plane and solid geometry, algebra beyond quadratics, plane trigonometry, and college algebra, with suggestions on methods of teaching each.

KENTUCKY.

Manual and Course of Study for County High Schools. Frankfort, 1911. 43 p.

Contains general suggestions on aim and organization of high schools, qualifications of teachers, and courses of study, with special suggestions on methods in algebra, geometry, trigonometry, arithmetic, and commercial arithmetic.

LOUISIANA.

State Course of Study. Baton Rouge, 1912. 158 p.

Supplement to state course of study and revision thereof. 27 p.

Outlines of courses, suggestions on methods, lists of authorized texts and reference books.

MARYLAND.

Course of Study for High Schools. Annapolis, 1913.

MASSACHUSETTS.

1. Memorandum on Program of Small High Schools. David Snedden, Boston, 1912. 8 p.

   Suggested revisions of programs in view of the fact that present programs are too largely determined by college entrance requirements.

2. Mathematics in the High Schools of Massachusetts. J. W. McDonald, Boston, 1909. 35 p.

   Suggestions and criticisms on methods of teaching algebra and geometry. Reprinted from 73d report of Massachusetts Board of Education.

3. Relation of Massachusetts High Schools to Community Needs, with special reference to the demand for the so-called practical subjects. 30 p.

   Abstract from a report of a special committee of the Massachusetts council of education, 1908.

MICHIGAN.

State Education Department Bulletins as follows:

No. 7, 11 p. 1910, Lansing.

No. 34, 16 p. 1911, Lansing.
No. 45, 31 p. 1912, Lansing.
No. 46, 22 p. 1912, Lansing.
No. 47, 28 p. 1912, Lansing.
No. 48, 29 p. 1912, Lansing.

All are devoted to the teaching of arithmetic, with suggested problems and problem sources.

MINNESOTA.

Catalogue of State University contains outline of mathematics course for secondary schools.

MISSISSIPPI.

State Education Department Bulletin, No. 8, Part II, Jackson, 1913. 8 p.

Contains course of study for county agricultural high schools.

NEVADA.

Course of Study for the Public Schools of Nevada. Carson City, 1912. 113 p.

Contains lists of authorized texts, supplementary and library books, and detailed suggestions on arithmetic teaching.

NEW HAMPSHIRE.

State Course of Study for High Schools. Concord. (Not available for general distribution.)

NEW JERSEY.

1. Syllabus of Mathematics for Secondary Schools.
2. Manual for High Schools. Trenton, 1913. 48 p.

Contains course of study and suggested programs.

3. The Teaching of Elementary Arithmetic. Trenton, 1912. 68 p.

NEW MEXICO.

Circular of Information. Santa Fe, 1913. Pamphlet.

Requirements for teachers' certificates.

NEW YORK.

State Syllabus for Secondary Schools. Albany, 1910. 481 p.

Outlines requirements in advanced arithmetic, elementary algebra, intermediate algebra, plane and solid geometry, trigonometry, and advanced algebra.

NORTH CAROLINA.

1. North Carolina High School Bulletin.
2. Handbook for High School Teachers. Raleigh.

PENNSYLVANIA.

Manual for Borough and Township High Schools. Harrisburg, 1908. 59 p.

Outlines of courses, with suggestions on methods, and lists of reference books.

SOUTH CAROLINA.

High School Manual for Teachers. Columbia, 1911. 101 p.

Contains course of study, with special suggestions on educational values and methods, and lists of authorized text and reference books.

TEXAS.

State Course of Study for High Schools. Austin.
Outline for mathematics course suggested.

UTAH.

High School Circular. Salt Lake City, 1913. 101 p.
Contains suggested course of study, and methods, with lists of recommended text and reference books.

VIRGINIA.

Course of study for Public High Schools. Richmond, 1910. 8 p.
Outline of prescribed courses for high schools of the first, second and third grades.

WEST VIRGINIA.

1. High School Manual. Charleston, 1912. 70 p.
Contains course of study with detailed suggestions on methods in algebra, plane and solid geometry.
2. Report of State Supervisors of High Schools for 1911 and 1912. Charleston, 1912. 40 p.
Statistics of high schools of the first, second, and third grades, with lists of authorized texts.

WISCONSIN.

1. High School Manual. Madison.
Contains outlines of courses with suggestions on methods.
2. Algebra in the High School. H. L. Terry, Madison. 6 p.
Reprinted from School Science and Mathematics, Vol. 10, 1910.
3. Bulletin of Information, No. 12. E. B. Skinner, chairman. Madison, 1905. 15 p.
Report of a committee of the State Teachers' Association on a revision of the content of geometry for high schools. Gives the list of propositions in plane and solid geometry for a year course as selected by the committee.

Publications of the U. S. Bureau of Education, Washington, D. C.

Annual Reports of the Commissioner of Education, 1867–1912.

1873.	List of educational publications in 1873, 20 mathematical titles.	pp. 826–845.
1874.	List of educational publications in 1874, 32 mathematical titles.	pp. 888–913.
1876.	List of educational publications in 1876, 16 mathematical titles.	pp. 896–920.
1877.	List of educational publications in 1877, 34 mathematical titles.	pp. 618–633.
1878.	List of educational publications in 1878, 29 mathematical titles.	pp. 702–719.
1879.	List of educational publications in 1879, 28 mathematical titles.	pp. 722–744.

1880. List of educational publications in 1880, pp. 876–902, 36 mathematical titles.

1881. List of educational publications in 1881, pp. 792–828, 45 mathematical titles.

1882–83. List of educational publications in 1882–83, pp. 820–860, 78 mathematical titles.

1883–84. List of educational publications in 1883–84, pp. 881–930, 66 mathematical titles.

1885–86. List of educational publications in 1885–86, pp. 677–708, 44 mathematical titles.

Time given to mathematics in English schools, pp. 737–738.

1886–1887.

1. Detailed comparison of programs in French and American secondary schools, pp. 1010–1014.
2. Requirements in 21 colleges compared, pp. 631–641.

1888–89.

A comparison of the schools of the United States, Germany and France, pp. 32–75.

Contains typical courses of study in Prussian and French high schools, and the number of hours per week devoted to each subject.

1890–91.

1. Time devoted to mathematics in French schools, p. 119.
2. Time devoted to mathematics in Japanese schools, p. 243.

1892–93.

1. Report of the committee of ten on secondary school studies, pp. 1415–1448.

   Contains an abstract of the report on mathematical teaching in secondary schools (pp. 1426–1427).
2. The curriculum for secondary schools. W. T. Harris. Pp. 1457–1464.
3. List of papers and addresses considered in the volumes of Proceedings of the National Educational Association from 1870 to 1893, inclusive. Pp. 1514–1549.

   Classified by subjects and by authors.

1893–94.

1. German Bibliography of the history and methods of arithmetic. Matthaeus Sterner. Pp. 314–323.

   230 titles, all German.
2. Report of the committee of fifteen on correlation of studies. Pp. 497–502. Part B.

   Relates to arithmetic and the transition to algebra.
3. Educational bibliography. Pp. 1701–1722.

   Mathematical section, p. 1716, contains 7 titles.

1896–97.

Entrance requirements for engineering colleges. Pp. 891–898. Also published separately as No. 238–16.

Report of a committee of the Society for the Promotion of Engineering Education, August, 1896. Eighty-nine engineering colleges classified according to entrance requirements, principally on a mathematical basis. Uniform requirements recommended in arithmetic, algebra, plane and solid geometry and plane trigonometry.

1897–98.

Notes on the history of American textbooks in arithmetic. Pp. 789–868. Also published separately as No. 257–18. J. M. Greenwood and Artemas Martin.

Describes 250 American arithmetics from 1729 to 1860.

1898–99.

1. The U. S. Naval Academy at Annapolis. Edward S. Holden. Pp. 747–780. Also published separately as No. 216–13.
   Contains details of the mathematical course with textbooks.
2. Notes on the history of American textbooks in arithmetic. Pp. 781–835. Also published separately as No. 260–14. J. M. Greenwood and Artemas Martin.
   Describes 250 American arithmetics from 1861–1892.
3. Confederate Textbooks (1861–1865). Pp. 1139–1155. Also published separately as No. 260–20. Stephen P. Weeks.
   Bibliography and description, including 11 arithmetics.

1899–1900.

Statistical table giving number and percentage of pupils in public and private high schools, studying algebra, geometry and trigonometry. P. 2122.

(Similar information is given in nearly all the annual reports.)

1903.

1. Secondary Education. Pp. 553–583. Also published separately as No. 341–13. Elmer E. Brown.
   Gives courses of study of early American academies, and material on modern high school courses.
2. Education in France. Pp. 585–622. Also published separately as No. 341–14.
   Contains comparison of American and French secondary courses of study.
3. State Normal Schools of the United States. Pp. 1103–1136. Also published separately as No. 341–23. E. O. Lyte.
   Contains typical normal school courses of study.
4. Exhibit of Bureau of Education at the Louisiana Purchase Exhibition. Pp. 1137–1184. Also published separately as No. 341–24.
   Shows development of mathematical curriculum from seventeenth century.
5. Courses of Study in German Schools. Pp. 1217–1242. Also published separately as No. 341–27.

1904.

The Educational System of Sweden. Pp. 767–797. Also published separately as No. 356–10.

Shows amount of time given to mathematics in Swedish schools.

1905.

Report of the Mosely Commission to the United States of America, 1903. Pp. 11–38. Also published separately as No. 365–3.

1907.

Statistical tables giving number of pupils in public and private high schools studying algebra, geometry and trigonometry from 1889 to 1906. P. 1050.

1909.

American Federation of Teachers of Mathematical and Physical Sciences. P. 56.

Account of formation, constituent bodies, etc.

1910.

1. American Federation of Teachers of Mathematical and Physical Sciences. P. 54.
   Meeting held at Boston, 1909. Outline of committee report on geometry syllabus.
2. International Commission on the Teaching of Mathematics. Pp. 159–162.
   Account of organization, scope, general plan and progress of the inquiry in the United States.
3. Secondary Schools. Pp. 1127–1196.
   Contains course of study recommended by the committee of ten, and amount of time to be devoted to each subject, including algebra, geometry and trigonometry.

1911.

American Federation of Teachers of the Mathematical and Physical Science. P. 237.

Resolutions on college entrance examinations in mathematics, presented at the meeting of December 28, 1910.

Circulars of Information.

1. Publication No. 167. (Circular of Information, 1890, No. 3.) The Teaching and History of Mathematics in the United States. 400 p. Florian Cajori.

   An extensive and valuable work. (Now out of print.) Contents:

   Part 1. Colonial times.
   Part 2. Influx of English Mathematics.
   Part 3. Influx of French Mathematics.
   Part 4. Mathematical Teaching at the Time.
   Part 5. Historical Essays.
   (*a*) History of Infinite Series.
   (*b*) On Parallel Lines and Allied Subjects.
   (*c*) On the Foundations of Algebra.
   (*d*) Difference between Napier's and Natural Logarithms.
   (*e*) Circle Squarers.

Appendix contains a bibliography of Fluxions and the Calculus.

2. Publication No. 205. 244 p.

Report of the committee on secondary school studies appointed at the meeting of the National Educational Association, July 9, 1892, with the reports of the conferences arranged by this committee and held December 28, 1892.

Bureau of Education Bulletins.

1908, No. 2. 69 p.

List of Publications of the U. S. Bureau of Education, 1867–1907.

1910, No. 3, 55 p.

List of Publications of the U. S. Bureau of Education, 1867–1910.

1911, No. 4. 27 p.

Bibliography of Science Teaching.

Compiled by a committee of the American Federation of Teachers of Mathematics and Science. Mathematics Section contains 40 titles.

1911, No. 6. 63 p.

Graduate Work in Mathematics in Universities and in other Institutions of like grade in the United States.

Report of Committee No. XII, Maxime Bôcher chairman, of the International Commission on Mathematical Teaching. (American Report.) Contains the general report, and that of subcommittee 1 on Courses of Instruction and the Master's Degree; of subcommittee 2 on Preparation for Research and the Doctor's Degree; and of subcommittee 3 on the Preparation of College and University Instructors in Mathematics.

1911, No. 7. 30 p.

Undergraduate Work in Mathematics in Colleges of Liberal Arts and Universities.

Report of Committee No. X, H. S. White chairman, of the International Commission on Mathematical Teaching. Contains, besides the general report, reports of subcommittees on men's colleges, on women's colleges, and on co-educational colleges.

1911, No. 8. 72 p.

Examinations in Mathematics other than those set by the teacher for his own Classes.

Report of Committee VII, T. S. Fiske chairman, of the International Commission on Mathematical Teaching. Contains the general report and that of subcommittees as follows:

1. Nature of Promotion in Elementary Schools and Admission to Secondary Schools.
2. Entrance to College by College Examinations.
3. Entrance to College by State Examinations.
4. Entrance to College by Entrance-board Examinations.
5. Entrance to College by Certification.
6. State and Local Examinations of Teachers.
7. Examinations of Actuaries.

1911, No. 9. 44 p.

Mathematics in the Technological Schools of Collegiate Grade in the United States.

Report of Committee No. IX, H. W. Tyler chairman, of the International Commission on Mathematical Teaching. Contains the general report and reports of subcommittees on independent technological schools, and on technological departments of colleges and universities.

1911, No. 12. 23 p.

Training of Teachers of Elementary and Secondary Mathematics.

Report of Committee No. V, E. H. Taylor chairman, of the International Commission on Mathematical Teaching. (American Report.) The report is made up as follows:

1. The Training of Teachers of Mathematics in professional schools of collegiate grade, separate from or connected with colleges or universities.
2. State Normal Schools.
3. Private Normal Schools.
4. Teachers for Normal Schools.

1911, No. 13. 185 p.

Mathematics in the Elementary Schools of the United States.

Report of Committees I and II of the International Commission on Mathematical Teaching. (American Report.)

1911, No. 16. 187 p.

Mathematics in the Public and Private Secondary Schools in the United States.

Reports of Committees III and IV, C. W. Evans and Wm. E. Stark chairmen, of the International Commission on Mathematical Teaching. (American Report.) Contains the general reports and special reports as follows:

On Boys' High Schools.

On Girls' High Schools.

On Co-educational High Schools in the East, in the West, in the South, and on the Pacific Coast.

On Preparation of Teachers of Mathematics for Public High Schools.

On the Six Year High School.

On Failures in the Technique of the Teaching of Secondary Mathematics—Their Causes and Remedies.

The report of Committee IV on Mathematics in Private Secondary Schools discusses Aim and Methods, Examinations and Tests, Syllabus Method in the Polytechnic Preparatory School of Brooklyn, Mixed Mathematics in the University High School, Chicago, Mathematical Club in the Shattuck School, Faribault, Minn., etc. In appendix A, Mathematical Instruction in the Evening Technical Schools; B, Mathematical Teaching in Private Correspondence Schools; C, Teaching of Mathematics in Schools and Colleges for Negroes.

1912, No. 2. 25 p.

Mathematics at West Point and Annapolis.

Report of Committee XI, C. B. Upton chairman, of the International Commission on Mathematical Teaching, contains special reports as follows:

1. Mathematics in the Training of Army Officers, including schools for graduates of West Point. Includes U. S. Military Academy; Engineer School of Application, Washington; Ordnance School of Application, Sandy Hook; and Coast Artillery School.
2. Mathematics in Schools for Training Naval Officers, including school for graduates of Annapolis.

1912, No. 4. 35 p.

Mathematics in the Technical Secondary Schools of the United States.

Report of Committee VI, C. N. Haskins, chairman, of the International Commission on Mathematical Teaching. (American Report.) Contains special reports as follows:

1. On Public, Private, and Corporation Trade Schools (incorporated in the general report).
2. On Public and Private Commercial Schools.
3. On Agricultural Schools.
4. Supplementary Report on the Industrial School of Secondary and Intermediate Grade.

1912, No. 13. 47 p.

Influences Tending to Improve the Work of the Teacher of Mathematics.

Report of Committee No. VIII, E. P. Cubberly chairman, of the International Commission on Mathematical Teaching. (American Report.) Contains special reports as follows:

1. On Scientific Societies and Periodical Literature.
2. On Teachers' Associations.
3. On Teachers' Institutes.
4. On State Supervision and Inspection of Instruction.
5. On Activities of Publishers and their Agents.
6. On Teaching of Mathematics in Summer Sessions of Universities and Normal Schools.

1912, No. 14. 84 p.

Report of the American Commissioners of the International Commission on the Teaching of Mathematics.

1912, No. 29. 95 p.

Bibliography of the Teaching of Mathematics, 1900 to 1912.

David Eugene Smith and Charles Goldziher. 1849 titles, including both English and foreign.

1913.

Report of the National Committee of Fifteen on Geometry Syllabus (jointly authorized by the National Educational Association and the American Federation of Teachers of the Mathematical and the Natural Sciences).

PART III.

Association Reports and Publications and College Publications.

(Exclusive of those published through the educational journals.)

Topics: Mathematics in the Secondary Schools; The Teaching of Algebra; The Teaching of Geometry; The Teaching of Trigonometry.

Mathematics in the Secondary Schools.

1. Breckenridge, W. E. Applied Problems. Report of the Schoolmaster's Association of New York, 1910–1911.
2. Burger, C. R. What have we a Right to Expect of the High School in Mathematics Preparation for the College? Proceedings of the Colorada State Teachers' Association.
3. Chandler, E. F. Approximations as a Check upon Mathematical Calculations. 1912 Proceedings North Dakota Educational Association.
4. Comstock, C. E. Unification of Secondary Mathematics. Proceedings of National Education Association, 1904.
5. Evans, George W. The Teaching of Mathematics in the Public Secondary Schools of the United States. Report of the Schoolmaster's Association of New York, 1910–1911.
6. Feldman, D. D. Loss of Efficiency in the Recitation in Mathematics. Report of the Schoolmaster's Association of New York, 1912–1913.
7. Hart, W. W. Review of the Teaching of Mathematics in the Past Decade. Proceedings of Central Association of Science and Mathematics Teachers, 1912.
8. Mathematics in the High School. Bulletin 193, the University of Texas.
9. Moore, E. H. On the Foundations of Mathematics. Bulletin of the American Mathematical Society, Vol. 9, No. 8, May, 1903.
10. Mortiz, R. E. Mathematical Books for the High School Shelf. Special Bulletin of the State Library, Olympia, Washington.
11. Patterson, J. L. Economy in Mathematics Instruction. Proceedings National Education Association, 1896.
12. Report of the Committee of the American Mathematical Society of the Definitions of College Entrance Requirements. Bulletin, Vol. X, No. 2, November, 1903.
13. Report of the Committee on Entrance Requirements in Mathematics. Proceedings of the National Education Association, 1903.
14. Report of the Committee on Unifying Mathematics. Proceedings of the Central Association of Science and Mathematics Teachers, 1908.
15. Simcox, W. B. Cultural Mathematics versus Practical Mathematics. Proceedings North Dakota Educational Association, 1912.
16. Skinner, E. B. The High School Course in Mathematics. Bulletin of University of Wisconsin, 350.

17. ———. Some Desirable Modifications in the Course of Study for Secondary Mathematics. Bulletin No. 2, 1912, Wisconsin State Teachers' Association.
18. Smith, Eugene R. Recent Progress in Mathematics Teaching. Report of the Schoolmaster's Association of New York, 1910–1911.
19. Stark, W. E. The Teaching of Mathematics in the Private Secondary Schools of the United States. Report of the Schoolmaster's Association of New York, 1910–1911.
20. Syllabus of Mathematics Report compiled by the Committee on the Teaching of Mathematics to Students of Engineering, Society for the Promoting of Engineering Education. For sale by secretary, Ithaca.
21. Tinsley, S. B. Amount of Mathematics in the Secondary School Course. Proceedings of National Education Association, 1904.
22. Thornton, E. L. Mathematics in Commercial Work. Proceedings of National Education Association, 1903.

The Teaching of Algebra.

1. Algebra for Secondary Schools. Proceedings of National Education Association, 1909.
2. Mathematical Review, Part II. Algebra. Published by Rensselaer Polytechnic Institute.
3. Report of the Committee of the Central Association on Algebra in the Secondary Schools. The Central Association of Science and Mathematics Teachers.
4. Syllabus of Elementary and Intermediate Algebra. Association of Teachers of Mathematics in the Middle States and Maryland.

The Teaching of Geometry.

1. Benedict, H. Y., and Calhoun, J. W. The Teaching of Plane Geometry. Bulletin No. 243, the University of Texas.
2. Geometry and Alegbra Teaching in Secondary Schools. Proceedings of National Education Association, 1909.
3. Loci. Published by the Association of Mathematical Teachers in New England.
4. Mathematical Review. Part III. Geometry. Published by Rensselaer Polytechnic Institute.
5. Report of the Committee on Geometry. Proceedings of the Central Association of Science and Mathematics Teachers, 1908.
6. Report of the National Committee of Fifteen on Geometry Syllabus. See Bureau of Education, 1913.
7. Syllabus of Propositions in Geometry. Harvard University. First Edition, 1896, revised 1912.
8. Syllabus of Geometry. Edited by the Association of Mathematical Teachers of New England. Ginn & Co.

9. Travis, C. R. Humanizing Geometry. Proceedings of North Dakota Educational Association, 1913.

The Teaching of Trigonometry.

1. Moritz, R. E. Desiderata in Modern Courses in Plane Trigonometry. Proceedings of the Washington State Educational Association, 1907.
2. Trigonometry in the High School. Bulletin 208, the University of Texas.

TEACHING OF FACTORING.

By E. M. Dow.

When new subjects seek admission to the curriculum on equal terms with their elder sisters (or on any terms, for that matter), they are told that their subject matter has not been organized, tested and tried in the crucible of time, and hence their educational value is uncertain at best. In the case of mathematics great minds have been concerned with its teaching for hundreds of years and thousands of texts have put before students the principles and methods of the science until we moderns surely have a right to expect under Darwin's law of survival of the fittest that the methods and processes of the present day are the best up to date if not indeed perfect. And the progressive adaptation of mathematics and its teaching to the spirit of the times has been due in part to the great geniuses whose names we respect and whose works are imperishable; and partly to the rank and file of mathematics teachers who by intention or good fortune have hit upon a device that works and have made this common property for their fellows. So today I wish, as a little one in mathematics and not even as the original discoverer of the device, to bring you a scheme for factoring which "works" very well. I am perhaps only the mouthpiece for my colleagues who use the scheme with even more deadly effect than I do.

In speaking on this topic of factoring I shall assume that the texts bring out with sufficient clearness how a polynomial is analyzed into its factors, that in our days of abounding vitality and confidence when we feel that we are really teaching we preface our work in factoring with the special theorems in multiplication and make our students realize that we are at the task of undoing work that resulted in the expression before us; that we teach carefully with many examples how a monomial factor is spotted and ferretted out, how binomials with two squares and a minus sign or two cubes or two odd powers

and a sign are treated, how a trinomial yields to the analyst's hammer and how more imposing arrays of terms are conquered. In other words I shall assume that we teach our classes sufficient algebraic "materia medica" so they will normally apply the correct dose to a given case of factoring or will use the scalpel skilfully when the nature of the lesion is apparent.

To follow out the medical analogy just suggested, a doctor must know not only what medicine to apply to a given disease but must also be able to decide what disease is present in a patient before him: his diagnosis precedes any application of healing remedies. When confronted with certain symptoms he must say to himself "Now this sore throat may be from tonsilitis, quinsy, diphtheria, etc., this fever from several sources, etc.," and each added symptom helps him to reach a correct decision on the exact nature and name of the disease. Then his training in dosage and pillage sends his fingers to a suitable bottle in his satchel. Isn't diagnosis the more difficult part to practise with security? And doesn't the medical school lay emphasis on both diagnosis and prescribing?

To take another analogy and look at troubles through the eyes of a wire chief in a telephone exchange. When a subscriber sends a complaint the first duty of the wire chief is to find out what sort of trouble is present, and that often takes rather extended search. He knows what to do for almost any form of short circuit, wet cord or crossed wires, loose connections or broken poles, but he doesn't get out tools for any one of them until he finds out what the trouble is and what tools he can use to advantage.

Again a salesman for either specialties or staples must know his catalogue pretty well in order to sell the goods his customer needs or wants. And if a lady asks to see some dress goods, the clerk must find out whether she means silk or broadcloth or calico and can then take down the right kind.

In all these cases I have tried to show that diagnosis or "sizing up" is fully as important as the determined action which follows or accompanies; that often the required action is brief compared to the study of conditions; and the instances cover a fairly wide range of experience. Can we then put our students in the place of the algebraic doctor, wire chief or salesman and

make them feel that the work of factoring proceeds along the same track as the mental work in the operations of real life? Can we stimulate them to the analysis of a polynomial by making them feel that they are thus living out in imagination at least their favorite occupation?

To come at length to the scheme for factoring which I suggested at first as being effective. The boy or girl must be taught a catalogue or list of algebraic symptoms or styles and the appropriate action associated with each name in this list. And this catalogue to reduce the labor of diagnosis to a minimum and yet make it certain must be logical, simple and catchy or easily remembered.

CATALOGUE OR SCHEME FOR FACTORING.

I. *Monomial Factor.*
$aQ + bQ - cQ.$

II. *2-terms (or Binomials).*
$Q^2 - O^2$
$Q^3 \pm O^3$
$Q^5 \pm O^5$

III. *3-terms (or Trinomials).*
$Q^2 \pm 2QO + O^2$
$Q^2 + (a + b)Q + ab$
$aQ^2 + bQ + c$
$Q^4 \pm mQ^2O^2 + O^4$

IV. *4-terms.*
$aQ + bQ + aO + bO$
$Q^2 + 2QO + O^2 - a^2$

V. *Factor Theorem.*

If we let a big round Q stand for a quantity then if we propose to look for a monomial factor we should expect to find a Q in each term and what would be a better name for this kind in our catalogue than "$aQ + bQ - cQ$"? If we frankly call it by this name we thus give it distinction and at the same time suggest the course to be followed in factoring it.

If we wish next to characterize the style with two squares and a minus sign we say this is $Q^2 - O^2$. We may notice at

this point that the Q and O were originally parentheses which have closed at top and bottom and thus acquired simpler and better known names, *e. g.*,

$$(a+b)^2-(c+d)^2=\boxed{a+b}^2-\boxed{c+d}^2.$$

And so on through the list of types which is here before us. They are arranged in order of difficulty and if a student tries the cases in this order he will generally obtain more quickly the correct diagnosis even if he has to discard two or three of the early trials. I have added the factor theorem as a last resort when the regular types fail though it is rather a luxury than a necessity.

It's a great scheme and its essentials are: (1) using Q and O and (2) trying each case in the order given, the former being the basis of its claim to novelty and effectiveness. The order is logical, but not necessarily pedagogical and the different cases can be taken up in any convenient order so long as the whole scheme is in the mind of the teacher and completed as a summary at the end. If your text develops the subject in this order well and good or if you take liberties with your text as an expert is entitled to you will find the order not only logical but simple, striking and easily remembered. Most books leave the teacher free to draw up a summary for himself. So if you refer to this list as the catalogue or menu and make your students feel that in this algebraic diagnosis they are being trained for the more vital sizing up of conditions in real life it will help to fasten the matter in the mind of the youthful and imaginative factorizer and tend to give his brain speed and his hand skill to perform the few operations that are so simply done when once decided on correctly.

MECHANIC ARTS HIGH SCHOOL,
BOSTON.

NEW BOOKS.

New Plane Geometry. By EDWARD RUTLEDGE ROBBINS. New York: American Book Company. Pp. vi + 264.

This text covers a maximum course, having two hundred formally proved theorems, many of which might well be omitted or relegated to the appendix. The first book seems very much confused in its order, as theorems on triangles, parallels, quadrilaterals and locus are intermingled with no evident division to fix the order for the pupil. It closes with a helpful classified summary of the method discovered for future use. This idea is not followed out throughout.

The section on variables and limits is open to grave criticism as to its correctness. It would be better to omit proofs rather than to beg the question. In the work on locus the author uses the "negative" method of proof.

Among the good features of the book are its attractive pages, its easily read proofs and some good sets of exercises.

Descriptive Geometry. By H. W. MILLER. New York: John Wiley and Sons. Pp. 149. $1.50.

This is the third edition of this book. It is written in simple conversational style and shows itself to be the work of a careful teacher. The problems are taken up by "Discussion," "Analysis," and "Construction," and the aim is evidently, as is made clear in the preface, the development of the student's power to think analytically. The reference chapter at the beginning, and the "Quiz Sheets" at the end, should both prove valuable.

First Year Course in General Science. By CLARA A. PEASE. New York: Charles E. Merrill Company. Pp. 315. $1.20.

The book offers a foundational course in science, including parts of physical geography, astronomy, biology, physics, chemistry and geology. It prepares the pupil for later specialization, rather than attempting to finish any subject, but it is a book for serious study, not one for casual reading. It has good type, interesting subject matter and many illustrations, and seems well worth examination.

Plane Trigonometry. By A. M. HARDING and J. S. TURNER. New York: G. P. Putnam's Sons. Pp. xiii + 158; Tables 51. Without tables 90 cents, with, $1.10.

The first chapter reviews the geometry used in trigonometry, giving the proofs as well as the principles. The authors have tried to keep the pupil from feeling lost in the new subject by developing the practise

before taking up much of the theory. Triangles are finished with as little theory as possible, some necessary relations being assumed subject to later proof, and the more abstract theoretical work all follows the part on triangles. There are numerous interesting features, including some excellent applications of trigonometry to geometry, and some rather uncommon proofs. The formulas to be memorized are emphasized by black type, but there is no distinction between those really important and some that could as well be omitted. The tables are conveniently arranged five place tables in type that, although rather small, is still easy to read.

A Review of High-School Mathematics. By WILLIAM D. REEVE and RALEIGH SCHORLING. Chicago: The University of Chicago Press. Pp. x + 70. 40 cents net.

This book contains the material used for review at the University High School of the University of Chicago. It covers algebra, geometry, and some of the elements of trigonometry. There is an abundance of good material which is sometimes arranged so that it serves to group the ideas in preparation for future use, but at other times seems too much influenced by the teaching order used in first going over the subjects. In all probability this results from the fact that the book is planned to serve the double purpose of a review book for each year, and a final fourth year review of all college entrance mathematics. At the end of the book the authors give their idea of a minimum course in mathematics for the first year and a half.

Education Through Concrete Experience. Volume IV of the Francis W. Parker School Year Book. Chicago: Press of The Francis W. Parker School. Pp. 186. 35 cents.

This book has been written by the faculty of the school to illustrate their use of the concrete in the various departments. It contains such articles as "Mental Imagery in Geography," "The School Museum," "School Heating and Ventilation—A Study in Applied Physics," and "A History Newspaper." Of special interest to teachers of mathematics are "The Pupil's Experience as the Source of his Problems in Arithmetic" and "Experience Building in the Teaching of Geometry." The book is full of suggestion to both executive and teacher, and it merits a wide circulation.

Elementary Algebra. By H. E. SLAUGHT and N. J. LENNES. Boston: Allyn and Bacon. Pp. x + 357.

This book is planned to cover the first year in the subject. It gives a long course for this time, for it includes all the required topics for Elementary and Intermediate Algebra except the Progressions. Like previous books by these authors there is great emphasis on simple presentation and easy gradation in each topic, and on the side of concrete

applications. There is an abundance of exercises and problems through the text, and the last thirty-five pages is given up to review exercises on the various chapters. While it is true that the large part of the problems are concerned with concrete things, rather than having a practical application in themselves, there are many interesting ones, some of which correlate this subject with other school subjects to good purpose. The historical notes are well chosen and are attractively arranged. The table of contents and the index are particularly usable, and the book is attractive in its general makeup.

Thoughts on Ultimate Problems. By F. W. FRANKLIN. London: David Nutt. Pp. 150. 1/6.

The following topics from the table of contents gives an idea of the field covered: A synoptic statement of two Theodocies; Notes on a new theory of time; Altruism and happiness; Theory of discrete manifolds; Historical data concerning the birth of Christ; The Johannine problem; etc.

The author is a profound thinker and gives here a condensed statement of his conclusions in rather technical language.

Analytic Mechanics. By JOHN ANTHONY MILLER and SCOTT BARRETT LILLY. Boston: D. C. Heath & Co. Pp. 297.

This book treats of those fundamental principles of mechanics which they believe to be essential to the progress of a student pursuing physics, engineering or celestial mechanics.

One purpose of the work is to develop facility in the application of mathematical principles to physical phenomena and to this end there are a large number of problems throughout the book. For the most part the illustrative problems are drawn from real structures or real machines.

A Text-Book on Practical Mathematics for Advanced Technical Students. By H. LESSLIE MANN. London and New York: Longmans, Green & Co. Pp. 487. $2.10 net.

This book is intended to cover a two- or three-years' course and comprises algebra, trigonometry, calculus, and the application of these to concrete examples. By treating the different subjects in one book in this way the author can emphasize those topics most used in the work of the calculus and its applications. The problems are well chosen and a student who completes the book should have a good knowledge of the subjects treated.

Ten Years at Yale. A Series of Papers on Certain Defects in the University World of To-day. By GEORGE FREDERICK GUNDELFINGER. New York: The Shakespeare Press. Pp. 216. $1.00 net.

"This is a series of papers constructed from thoughts, remarks and observations, jotted down by the author from time to time during his

experiences at the university." The following are some of the topics treated: A defence of pessimism; The evil of tutoring; The folly of research; The decline of teaching; The art of bluffing; The disrespect for scholarship; Fraternity and idol-worship; The failure to educate.

Even though many readers may not agree with the author, they will undoubtedly find much to keep the interest and engage the thought in these times of unrest and dissatisfaction with some phases of college life.

NOTES AND NEWS.

THE ROCHESTER SECTION has held its two regular meetings during last year. The first was held at The University of Rochester on November 21, 1914, the program being as follows: "The Congruence Theorems and their Practical Application," H. Carlisle Taylor, East High School, Rochester; "The Classroom Presentation of Variables and Limits," Miss Eunice M. Pierce, Lockport; "Heronian Triangles," William P. Durfee, Hobart College, Geneva; "Freshman Mathematics in College—Possible Readjustments and their Effect on the High Schools," Arthur S. Gale, The University of Rochester; "Question Box and Informal Discussion," William Betz, East High School, Rochester.

The officers for the year were elected as follows: *Chairman,* Miss Mary M. Wardwell Hutchinson Central High School, Buffalo; *Secretary,* Arthur S. Gale, The University of Rochester; *Executive Committee,* in addition to the Chairman and Secretary *ex-officio,* Harry N. Kenyon, East High School, Rochester; Henry J. Lathrop, Brockport State Normal School; George G. McEwen, Fredoia State Normal School. The attendance was twenty-two.

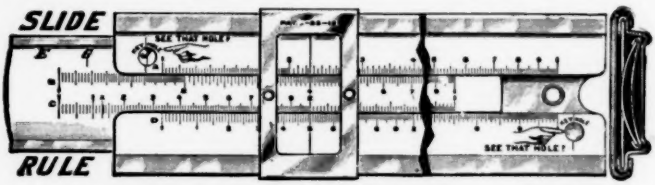
The second meeting was held at the Hutchinson Central High School, Buffalo, on May 8, 1915. About fifty were present and participation in the discussion of the papers was unusually general. The papers presented were as follows: "High School Mathematics," George G. McEwen, Fredonia State Normal School; "What the Supervising Officer Expects of the Teacher of Mathematics," Alfred C. Thompson, Brockport State Normal School; "A Discussion of an Important Influence Affecting a Student's Line of Thought," Charles J. Costello, Technical High School, Buffalo; "Elementary Algebra from the Functional Standpoint," Charles W. Watkeys, The University of Rochester.

Four applications for membership have been received during the year. The section voted to accept the invitation of Dr. Thompson, principal of the Brockport State Normal School, to hold the next meeting at Brockport. It will probably be held on November 6. It was also voted to refer to a committee certain questions on which the section may wish to petition the regents of the state of New York.

WHEELER'S "Examples in Algebra" has been adopted in quite a large number of important high schools.

LOVERS of rural life who have not already done so should read the editorial in *Farm Life* for June, 1915.

Something New in Slide Rules



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